



## A Research Report

A study and development of Windows based program for  
structural reliability analysis

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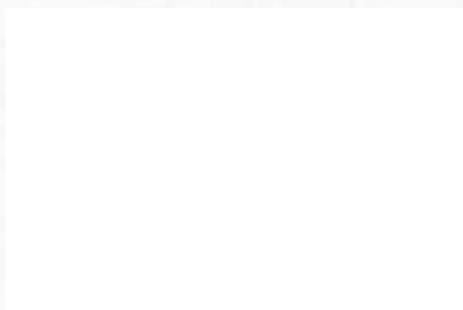
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### Abstract

In this report, a Windows application names "RAP" (Reliability Analysis Program) has been developed to evaluate the reliability of the structure, with known analytical expressions, which can be indicated in terms of reliability index and the probability of failure. In the computation part of the program, the reliability methods which are used in the program are First Order Reliability Method (FORM) and Monte-Carlo Simulation method (MCS). The result of reliability analysis using RAP program is compared with the one from the VaP program. The comparison shows that the result from the RAP program is almost the same as the one from the VaP in both methods.



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# Chapter 1

## Introduction

Structural risk and reliability analysis is the probabilistic engineering approach, also known as the "Reliability method", for supporting the structural integrity analysis of structures and components under service loads. The mathematical basis for this reliability method was established in the late sixties and early seventies (Payne, 1972). However, due to its requirement for high computational power and what was then unavailable data this method was unattractive for use in structural integrity analysis of structures. As a result the damage tolerance method, which was deterministic, was selected rather than the probabilistic method (Lincoln, 1972).

Nowadays, a new challenge has arisen to maintain structures service life longer in an environment. As a result the deterministic approach were not adequate. Furthermore advanced approach is required. For these reasons, the probabilistic approach is begin to gain new interests in structural design.

Structural reliability and probabilistic methods have continued to develop a growing importance in modern structural engineering curricula across the world. They are currently used in the development of new generation design codes, evaluation of existing structures and probability risk assessment.

In recent day, there are many commercial and research softwares developed to help engineer to perform the reliability analysis of the structure, including VaP (Petschacher et al., 1992), CALREL (UC Berkeley, 1989), COSSAN (University of Innsbruck, 2003), PROBAN



(DNV Software, 2003), ISPUD (University of Innsbruck, 1997), NESSUS (Southwest Research Institute, 2003) and STRUREL (Reliability Consulting Programs, GMBH, 2003). NESSUS is capable of both stochastic finite element and boundary analyses for structural simulations. Most of these programs are able to conduct Monte Carlo simulations and structural reliability analyses. However, the commercial packages are too expensive in Thailand, as such, the objective of this research is to implement a user-friendly computer program which would determine the reliability index and the probability of failure of given problems with known analytical expressions. The program developed in this report will be discussed in details in chapter three.

The next chapter of this report will provide the reader with the theoretical background needed to properly understand the subsequent development. Computer implementation will be discussed in chapter three, and numerical examples in chapter four. Finally, the report concludes with a chapter which summarizes the work and proposes natural extensions to the project.

## Chapter 2

# Theoretical Background in Reliability Analysis

This chapter will provide the essentials of reliability based analysis which most part are adapted from Ang and Tang (1984). First, the basic definition of the reliability index is mentioned after that the two distinct methods to evaluate it are presented.

### 2.1 Reliability Index Definition

The Performance Function  $F$  is a function which determines the performance or the state of the system. In general  $F$  is a function of one or more variables  $x_i$  which describe the geometry, material, loads, and boundary conditions

$$F = F(x_i) \quad (2.1)$$

and thus  $F$  is a random variable with its own probability distribution function, Fig. 2.1. A performance function evaluation typically require a structural analysis, this may range from a simple calculation to a detailed finite element study.

Reliability indices,  $\beta$  are used as a relative measure of the reliability or confidence in the ability of a structure to perform its function in a satisfactory manner. In other words they are a measure of the performance function.

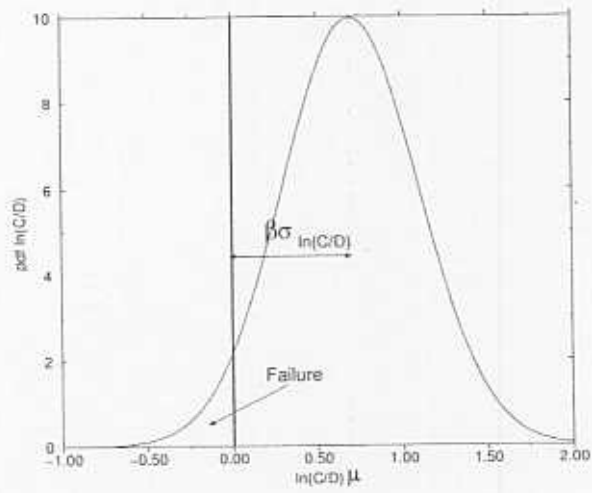
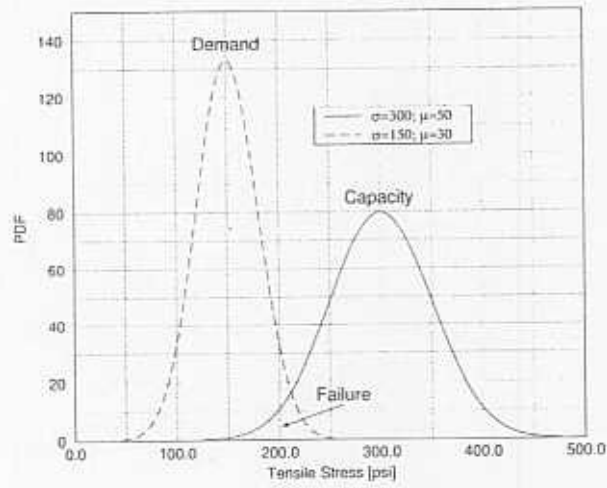


Figure 2.1: Definition of Reliability Index

Probabilistic methods are used to systematically evaluate uncertainties in parameters that affect structural performance, and there is a relation between the reliability index and risk.

Reliability index is defined in terms of the performance function capacity  $C$ , and the applied load or demand  $D$ . It is assumed that both  $C$  and  $D$  are random variables.

The safety margin is defined as  $Y = C - D$ . Failure would occur if  $Y < 0$ . If  $C$  and  $D$  are normal random variables with probability density function  $N(\mu_C, \sigma_C)$  and  $N(\mu_D, \sigma_D)$  respectively.  $Y$  is also a normal random variable with the probability density function  $N(\mu_Y, \sigma_Y)$  in which,

$$\mu_Y = \mu_C - \mu_D \quad (2.2)$$

$$\sigma_Y = \sqrt{\sigma_C^2 + \sigma_D^2} \quad (2.3)$$

and the reliability index  $\beta$  can be determined by the following

$$\beta = \frac{\mu_Y}{\sigma_Y} \quad (2.4)$$

$$= \frac{\mu_C - \mu_D}{\sqrt{\sigma_C^2 + \sigma_D^2}} \quad (2.5)$$

If  $C$  and  $D$  are lognormal random variables with means and standard deviations  $\mu_C$ ,  $\sigma_C$  and  $\mu_D$ ,  $\sigma_D$ , the corresponding parameters of the lognormal distribution are given by,

$$\lambda_C = \ln \mu_C - \frac{1}{2}\zeta_C^2; \quad \lambda_D = \ln \mu_D - \frac{1}{2}\zeta_D^2 \quad (2.6)$$

$$\zeta_C = \sqrt{\ln \left( 1 + \frac{\sigma_C^2}{\mu_C^2} \right)}; \quad \zeta_D = \sqrt{\ln \left( 1 + \frac{\sigma_D^2}{\mu_D^2} \right)} \quad (2.7)$$

In this case, the factor of safety which is defined as  $Y = \frac{C}{D}$  will be used and the failure event would be the event of  $Y < 1$ . Therefore,  $Y$  is also a lognormal variable with following

parameters.

$$\lambda_Y = \lambda_C - \lambda_D \quad (2.8)$$

$$\zeta_Y^2 = \sqrt{\zeta_C^2 + \zeta_D^2} \quad (2.9)$$

Then the reliability index is determined as follows

$$\beta = \frac{\lambda_Y}{\zeta_Y} \quad (2.10)$$

The probability of failure  $P_f$  is equal to the ratio of the shaded area to the total area under the curve in Fig. 2.1. For standard distributions and for  $\beta = 3.5$ , it can be shown that the probability of failure is  $P_f = \frac{1}{9,091}$  or  $1.1 \times 10^{-4}$ . That is one in every 10,000 structural members designed with  $\beta = 3.5$  will fail because of either excessive load or understrength sometime in its lifetime. Reliability indices are a relative measure of the current condition and provide a qualitative estimate of the structural performance. Structures with relatively high reliable indices will be expected to perform well. If the value is too low, the structure may be classified as a hazard. Target values for  $\beta$  are shown in Table 2.1, and in Fig. 2.2

Table 2.1: Selected  $\beta$  values for Steel and Concrete Structures

Expected Performance	$\beta$	Failures
High	5	3/10 million
Good	4	3/100,000
Above Average	3	1/1,000
Below Average	2.5	6/1,000
Poor	2.0	2.3/100
Unsatisfactory	1.5	7/100
Hazardous	1.0	16/100

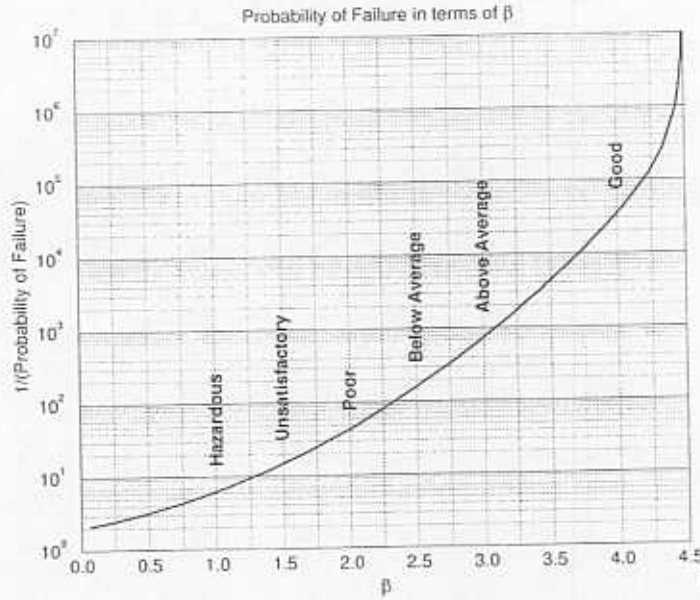


Figure 2.2: Probability of Failure in terms of  $\beta$

## 2.2 Monte-Carlo Simulation(MCS)

Monte Carlo simulation is a stochastic technique used to solve mathematical problems. The word "stochastic" means that it uses random numbers and probability statistics to obtain an answer.

In Monte Carlo simulation, the random selection process is repeated many times to create multiple scenarios. Each time a value is randomly selected, it forms one possible scenario and solution to the problem. Together, these scenarios give a range of possible solutions, some of which are more probable and some less probable.

When repeated for many scenarios [10,000 or more], the average solution will give an approximate answer to the problem. Accuracy of this answer can be improved by simulating more scenarios. In fact, the accuracy of a Monte Carlo simulation is proportional to the square root of the number of scenarios used.

### 2.2.1 Methodology

For general problem in engineering system, the capacity and demand maybe functions of the other random variables. In term of safety of margin, the performance function can be expressed as

$$g(C_1, C_2, \dots, C_n, D_1, D_2, \dots, D_n) = C(C_1, C_2, \dots, C_n) - D(D_1, D_2, \dots, D_n) \quad (2.11)$$

The failure state will occur when  $g < 0$ . The reliability index can be calculated by using Monte-Carlo Simulation as the follows:

1. Initialize random number generators,
2. Perform  $n$  analysis, for each one
  - (a) For each variable, determine a random number for the given distribution,
  - (b) Determine the performance function, and
  - (c) Analyze, and store the results,
3. Count the number of analyses,  $n_f$  which performance function indicate failure, the likelihood of structural failure will be  $p_f = n_f/n$ , and
4. The reliability index is then determined by using the following equation.

$$\beta = \Phi^{-1}(1 - p_f) \quad (2.12)$$

### 2.2.2 Generating of Random Numbers

Major key in the application of Monte-Carlo Simulation is the generation of the appropriate random numbers for the given distributions of the random numbers. For each random variable, the generation process can be done by the following steps:

1. Generate a uniformly distributed random number between 0 and 1.0, and

2. Use the inverse transformation method to transform the uniformly distributed random number to the corresponding random number with the given distribution.

For the inverse transformation method, it can be shown graphically in Fig. 2.3.

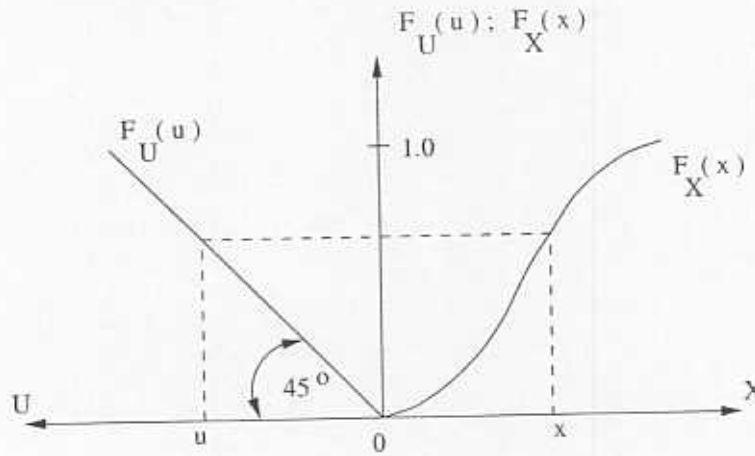


Figure 2.3: Inverse Transformation Method

From Fig.2.3, suppose  $U$  is a standard uniform variate with a uniform PDF between 0 and 1.0 and  $X$  is a random variate with its CDF  $F_X(x)$ .

If  $u$  is a value of the variate  $U$ , the cumulative probability of  $U \leq u$  is equal to  $u$ , Fig.2.4.

$$F_U(u) = u \quad (2.13)$$

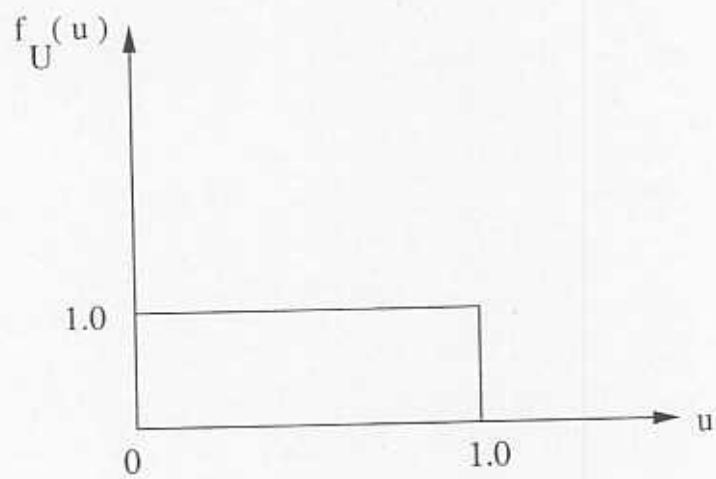
Therefore, For the variate  $X$ , at the cumulative probability  $u$ , the value of  $X$  can be calculated from

$$x = F_X^{-1}(u) \quad (2.14)$$

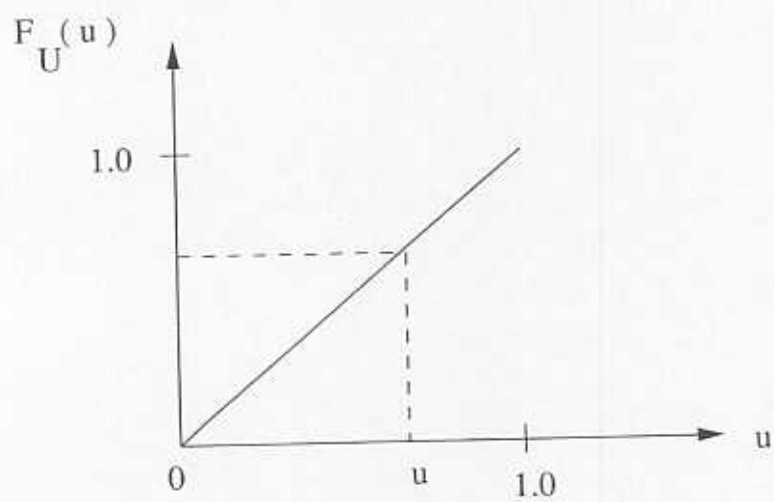
which means if  $(u_1, u_2, u_3, \dots, u_n)$  is the set of values from  $U$ , the corresponding set of values of  $(x_1, x_2, x_3, \dots, x_n)$  from  $X$  is obtained from

$$x_i = F_X^{-1}(u_i) \quad (2.15)$$





(a) PDF of  $U$



(b) CDF of  $U$

Figure 2.4: PDF and CDF of standard uniform variate

This transformation method can be used most effectively when the inverse of CDF of the random variable  $X$  can be expressed analytically.

## 2.2.3 Continuous Random Variables

As indicated in Section 2.2.2, the random numbers with a given non-normal distribution can be transformed through Eq. 2.14 once the standard uniformly distributed random numbers have been obtained by using the inverse transformation method. In this section, all distributions that are used throughout this report are considered.

### 2.2.3.1 Random Numbers with Shifted-Exponential Distribution

Considering the shifted-exponential distribution with CDF

$$F_X(x) = 1 - e^{\lambda(x-x_0)}; \quad x \geq x_0 \quad (2.16)$$

The inverse of this function is

$$x = F_X^{-1}(u) = -\frac{\ln(1-u)}{\lambda} + x_0 \quad (2.17)$$

Therefore, once the standard uniformly distributed random number  $u_i, i = 1, 2, 3, \dots, n$  are generated, we obtain the corresponding shifted exponentially distributed random numbers as

$$x_i = -\frac{\ln(1-u_i)}{\lambda} + x_0; \quad i = 1, 2, 3, \dots, n \quad (2.18)$$

Since  $(1-u_i)$  is also uniformly distributed, the required random numbers may also be generated by

$$x_i = -\frac{\ln(u_i)}{\lambda} + x_0; \quad i = 1, 2, 3, \dots, n \quad (2.19)$$

### 2.2.3.2 Random Numbers with Weibull Distribution

Considering the Weibull distribution with corresponding CDF

$$F_X(u) = 1 - \exp\left[-\left(\frac{x-\epsilon}{w_1-\epsilon}\right)^k\right] \quad (2.20)$$

The inverse function is

$$x = \epsilon + w_1(-\ln(1-u))^{\frac{1}{k}} - \epsilon(-\ln(1-u))^{\frac{1}{k}} \quad (2.21)$$

In this case, from this relation, the random numbers with Weibull distribution can be calculated once the uniformly distributed random numbers  $u$  are obtained. Since  $(1-u)$  is also uniformly distributed, the required random numbers may also be calculated from

$$x = \epsilon + w_1(-\ln(u))^{\frac{1}{k}} - \epsilon(-\ln(u))^{\frac{1}{k}} \quad (2.22)$$

### 2.2.3.3 Random Numbers with Asymtotic Type I (largest) Distribution

Considering the Asymtotic Type I (largest) distribution with corresponding CDF

$$F_X(u) = \exp[-e^{-\alpha(x-u_n)}] \quad (2.23)$$

The inverse function is

$$x = \frac{\alpha u_n - \ln(-\ln(u))}{\alpha} \quad (2.24)$$

In this case, from this relation, the random numbers with Asymtotic Type I (largest) distribution can be calculated once the uniformly distributed random numbers  $u$  are obtained.

For the random numbers of normal, lognormal and shifted-lognormal distribution, since their cumulative distribution function cannot be expressed analytically, the inverse transformation method may not be effective. To generate the numbers of these kinds of distributions, the other method which is called "function of random variables method" is used. The basis

of this method is as follow.

#### 2.2.3.4 Function of Random Variables Method

Suppose a random variable  $X$  can be expressed in term of a function of other variables  $Y_1, Y_2, Y_3, \dots, Y_n$ ; that is

$$X = g(Y_1, Y_2, Y_3, \dots, Y_n) \quad (2.25)$$

and the values of  $Y_1$  through  $Y_n$  can be generated. Then, a value of  $X$  can be obtained from

$$x = g(y_1, y_2, y_3, \dots, y_n) \quad (2.26)$$

where  $y_1, y_2, y_3, \dots, y_n$  are the random numbers which have been generated for  $Y_1, Y_2, Y_3, \dots, Y_n$ .

#### 2.2.3.5 Random Numbers with Normal Distribution

Based on the function of random variables method, Box and Muller (1958) have been introduced their own method to generating the random numbers with standard normal distribution. The basic of this method is: if  $U_1$  and  $U_2$  are two independent standard uniform variates, then the following list.

$$S_1 = (-2 \ln(U_1))^{\frac{1}{2}} \cos 2\pi U_2 \quad (2.27)$$

$$S_2 = (-2 \ln(U_1))^{\frac{1}{2}} \sin 2\pi U_2 \quad (2.28)$$

constitute a pair of static independent standard normal variates.

which means if  $u_1$  and  $u_2$  are a pair of independent uniformly distributed random numbers, a pair of independent random number of a standard normal distribution  $N(0,1)$  can be

obtained by the following.

$$s_1 = \sqrt{-2 \ln u_1} \cos 2\pi u_2 \quad (2.29)$$

$$s_2 = \sqrt{-2 \ln u_1} \sin 2\pi u_2 \quad (2.30)$$

In this report, the function `gasdev( )` which was taken from Press et al. (1988) is used. This function will return a standard normally distributed number. It can be shown as the following.

```
#include "math.h" float gasdev(long *idum) Returns a nomally
distributed deviate with zero mean and unit variance, using
ran1(idum) as the source of uniform deviates. {
    float ran1(long *idum);
    static int iset = 0;
    static float gset;
    float fac,rsq,v1,v2;
    if(iset == 0) {
        do {
            v1 = 2.0*ran1(idum)-1.0;
            v2 = 2.0*ran1(idum)-1.0;
            rsq = v1*v1+v2*v2;
        }while(rsq >= 1.0 || rsq ==0.0);
        fac = sqrt(-2.0*log(rsq)/rsq);
        gset = v1*fac;
        iset= 1;
        return v2*fac;
    }else{
        iset = 0;
        return gset;
    }
}
```

Suppose a normal variate  $X$  with distribution  $N(\mu, \sigma)$ . To obtain the normally distributed random numbers  $x_i, i = 1, 2, \dots$  with mean  $\mu$  and standard deviation  $\sigma$ , we knew that the

relationship between the value of normal distribution and standard normal distribution can be expressed by

$$s_i = \frac{x_i - \mu}{\sigma} \quad (2.31)$$

where  $s_i$  is the standard normal distribution numbers. Therefore, the required random numbers can be generated by

$$x_i = \mu + \sigma s_i \quad (2.32)$$

#### 2.2.3.6 Random Numbers with Lognormal Distribution

For a lognormal variate  $X$  with parameter  $\lambda$  and  $\zeta$ . Since  $\ln X$  is a normal random variable with mean  $\lambda$  and standard deviation  $\zeta$ . If  $x'$  is a the generated number of a normal distribution  $N(\lambda, \zeta)$ , the corresponding random number with lognormal distribution with parameter  $\lambda$  and  $\zeta$  can be determined by the following.

$$x = e^{x'} \quad (2.33)$$

#### 2.2.3.7 Random Numbers with Shifted-Lognormal Distribution

For a shifted lognormal variate  $X$  with parameter  $\lambda$  and  $\zeta$ . Since  $\ln(X - x_0)$  is a normal random variable with mean  $\lambda$  and standard deviation  $\zeta$ . If  $x'$  is a the generated number of a normal distribution  $N(\lambda, \zeta)$ , the corresponding random number with shifted-lognormal distribution with parameter  $\lambda$  and  $\zeta$  can be determined by the following.

$$x = e^{x'} + x_0 \quad (2.34)$$

## 2.3 First Order Reliability Method(FORM)

### 2.3.1 Theory

We define a *performance function* or *state function* as:

$$g(X) = g(X_1, X_2, X_3, \dots, X_n) \quad (2.35)$$

where,  $X = (X_1, X_2, X_3, \dots, X_n)$  is a vector of basic variables of system and the function  $g(X)$  determines the performance of the system. Therefore, we define the *limit-state* function of the system as  $g(X) = 0$ . Thus,  $g(X) < 0$  corresponds to the "the failure state" and  $g(X) > 0$  corresponds to the "safe state".

If the joint probability distribution function (PDF) of the design variables  $X_1, X_2, X_3, \dots, X_n$  is  $f_{X_1, X_2, X_3, \dots, X_n}(X_1, X_2, X_3, \dots, X_n)$  then, the probability of safety state is

$$p_s = \int_{g(X) > 0} f_X(X) dX \quad (2.36)$$

and the one of failure is

$$p_f = \int_{g(X) < 0} f_X(X) dX \quad (2.37)$$

The evaluation of  $p_s$  or  $p_f$  is generally a formidable task. Considering the case in which the basic variables  $(X_1, X_2, \dots, X_n)$  are uncorrelated, we define the *reduced variates* as:

$$X'_i = \frac{X_i - \mu_{X_i}}{\sigma_{X_i}}, \quad i = 1, 2, 3, \dots, n \quad (2.38)$$

thus the limit state equation can be rewritten in terms of the reduced variates:

$$g(\sigma_{X_1} X'_1 + \mu_{X_1}, \dots, \sigma_{X_n} X'_n + \mu_{X_n}) = 0 \quad (2.39)$$

We observe from Fig. 2.5 that as the limit-state surface moves closer to the origin, the safe region decreases. Since the variables  $X_i$  can only be positive, it is then quite evident

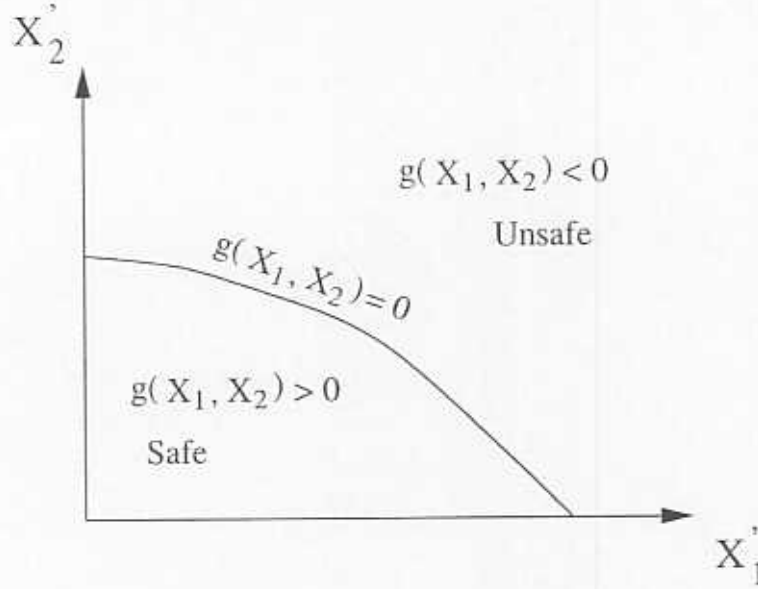


Figure 2.5: Safe states and failure states in space of reduced variates

that the position of the failure surface relative to the origin will determine the safety or *reliability* of the system. The position of the failure surface may be represented by the minimum distance from the surface  $g(X) = 0$  to the origin of the reduced variates. Thus, we can assume that this minimum distance is a measure of reliability and can be determined as follows. The distance from a point  $X' = (X'_1, X'_2, \dots, X'_n)$  on the failure surface  $g(X) = 0$  to the origin of  $X'$  is

$$D = \sqrt{X'^2_1 + \dots + X'^2_n} = (X'^t X')^{1/2} \quad (2.40)$$

and the point on the failure surface  $(x^*_1, x^*_2, \dots, x^*_n)$  which has the minimum distance to the origin may be determined by minimizing function  $D$ , subject to the constraint  $g(X) = 0$ ; that is,

Minimize  $D$  subject to  $g(X) = 0$



Using the method of Lagrange's multiplier, let

$$L = D + \lambda g(X) \quad (2.41)$$

$$= (X'^t X')^{1/2} + \lambda g(X) \quad (2.42)$$

$$= \sqrt{X_1'^2 + \dots + X_n'^2} + \lambda g(X_1, X_2, \dots, X_n) \quad (2.43)$$

where  $X_i = \sigma_{X_i} X_i' + \mu_{X_i}$

To minimize  $L$ , we obtain the following set of  $n + 1$  equations with  $n + 1$  unknowns:

$$\frac{\partial L}{\partial X_i'} = \frac{X_i'}{\sqrt{X_1'^2 + X_2'^2 + \dots + X_n'^2}} + \lambda \frac{\partial g}{\partial X_i'} = 0 \quad i = 1, 2, 3, \dots, n \quad (2.44)$$

$$\frac{\partial L}{\partial \lambda} = g(X_1, X_2, \dots, X_n) = 0 \quad (2.45)$$

The solution to the above set of equations should yield the most probable failure point  $(x_1^*, x_2^*, \dots, x_n^*)$ . We next introduce the gradient vector

$$G = \left( \frac{\partial g}{\partial X_1'}, \frac{\partial g}{\partial X_2'}, \dots, \frac{\partial g}{\partial X_n'} \right) \quad (2.46)$$

where

$$\frac{\partial g}{\partial X_i'} = \frac{\partial g}{\partial X_i} \frac{dX_i}{dX_i'} = \sigma_{X_i} \frac{\partial g}{\partial X_i} \quad (2.47)$$

thus, Eq. 2.44 can be written in matrix notation as

$$\frac{X'}{(X'^t X')^{1/2}} + \lambda G = 0 \quad (2.48)$$

from which

$$X' = -\lambda D G \quad (2.49)$$

Therefore,

$$D = [(\lambda D G^t)(\lambda D G)]^{1/2} = \lambda D (G^t G)^{1/2} \quad (2.50)$$

and thus

$$\lambda = (G^t G)^{-1/2} \quad (2.51)$$

Substituting into Eq. 2.49

$$X' = \frac{-GD}{(G^t G)^{1/2}} \quad (2.52)$$

Conversely (premultiplying Eq. 2.52 by  $G^t$ ),

$$D = \frac{-G^t X'}{(G^t G)^{1/2}} \quad (2.53)$$

Substituting Eq. 2.52 in to Eq. 2.45, results in a single equation with the unknown  $D$ ; solution of the resulting equation then yields the minimum distance  $d_{\min} = \beta$ ; thus

$$\beta = \frac{-G^{*t} X'^*}{(G^{*t} G^*)^{1/2}} \quad (2.54)$$

where  $G^*$  is the gradient vector at the most probable failure point  $(x_1^*, x_2^*, \dots, x_n^*)$ . In scalar form Eq. 2.54 is

$$\beta = \frac{-\sum_i x_i^* \left( \frac{\partial g}{\partial x_i} \right)_*}{\sqrt{\sum_i \left( \frac{\partial g}{\partial x_i} \right)_*^2}} \quad (2.55)$$

where the derivatives  $\left( \frac{\partial g}{\partial x_i} \right)_*$  are evaluated at  $(x_1^*, x_2^*, \dots, x_n^*)$ .

Using the above  $\beta$  in Eq. 2.52, the most probable point on the failure surface becomes

$$X'^* = \frac{-G^* \beta}{(G^{*t} G^*)^{1/2}} \quad (2.56)$$

and in scalar form, the components of  $X'^*$ , Eq. 2.56 are

$$x_i'^* = -\alpha_i^* \beta; \quad i = 1, 2, \dots, n \quad (2.57)$$

in which

$$\alpha_i^* = \frac{\left(\frac{\partial g}{\partial X_i}\right)_*}{\sqrt{\sum_i \left(\frac{\partial g}{\partial X_i}\right)_*^2}} \quad (2.58)$$

are the direction cosines along the axes  $x_i'$ . We observe that Eq. 2.54 and 2.56 can be interpreted on the basis of *first order approximation* for the function  $g(X)$ . i.e. the performance function is expanded in a Taylor series at point  $x^*$  which is on the failure surface  $g(x^*) = 0$ . That is,

$$\begin{aligned} g(X_1, X_2, \dots, X_n) &= g(x_1^*, x_2^*, \dots, x_n^*) + \sum_{i=1}^n (X_i - x_i^*) \left(\frac{\partial g}{\partial X_i}\right)_* \\ &\quad + \frac{\sum_{j=1}^n \sum_{i=1}^n (X_i - x_i^*)(X_j - x_j^*)}{\left(\frac{\partial^2 g}{\partial X_i \partial X_j}\right)_*} + \dots \end{aligned}$$

where the derivatives are evaluated at  $(x_1^*, x_2^*, \dots, x_n^*)$ . But  $g(x_1^*, x_2^*, \dots, x_n^*) = 0$  on the failure surface; therefore,

$$\begin{aligned} g(X_1, X_2, \dots, X_n) &= \sum_{i=1}^n (X_i - x_i^*) \left(\frac{\partial g}{\partial X_i}\right)_* \\ &\quad + \frac{\sum_{j=1}^n \sum_{i=1}^n (X_i - x_i^*)(X_j - x_j^*)}{\left(\frac{\partial^2 g}{\partial X_i \partial X_j}\right)_*} + \dots \end{aligned}$$

Recall that

$$\begin{aligned} X_i - x_i^* &= (\sigma_{X_i} X_i' + \mu_{X_i}) - (\sigma_{X_i} x_i'^* + \mu_{X_i}) \\ &= \sigma_{X_i} (X_i' - x_i'^*) \end{aligned}$$

and

$$\begin{aligned} \frac{\partial g}{\partial X_i} &= \frac{\partial g}{\partial X_i'} \left( \frac{dX_i'}{dX_i} \right) \\ &= \frac{1}{\sigma_{X_i}} \left( \frac{\partial g}{\partial X_i'} \right) \end{aligned}$$

Then

$$g(X_1, X_2, \dots, X_n) = \sum_{i=1}^n (X_i' - x_i'^*) \left( \frac{\partial g}{\partial X_i'} \right)_* + \dots$$

The mean value of the function  $g(X)$  in first order approximation is obtained by truncating the above series at the first order term. That is,

$$\mu_g \simeq - \sum_{i=1}^n x_i'^* \left( \frac{\partial g}{\partial X_i'} \right)_* \quad (2.59)$$

and the first order approximation of the variance is

$$\sigma_g^2 \simeq \sum_{i=1}^n \sigma_{X_i}^2 \left( \frac{\partial g}{\partial X_i'} \right)_*^2 \quad (2.60)$$

$$= \sum_{i=1}^n \left( \frac{\partial g}{\partial X_i'} \right)_*^2 \quad (2.61)$$

From Eq. 2.59 and 2.61, the ratio

$$\frac{\mu_g}{\sigma_g} = \frac{- \sum_{i=1}^n x_i'^* \left( \frac{\partial g}{\partial X_i'} \right)_*}{\sqrt{\sum_{i=1}^n \left( \frac{\partial g}{\partial X_i'} \right)_*^2}} \quad (2.62)$$

We can see that the above value is the same as Eq.2.55. Therefore the reliability index is

$$\beta = \frac{\mu_g}{\sigma_g} \quad (2.63)$$

The evaluation of the reliability index through first order approximation of the Taylor series expansion and through first and second moments of the random variables, is termed “First-Order Second Moment Reliability Method” or shortly First Order Reliability Method (FORM).

Considering the general case in which the performance function  $g(X)$  is nonlinear, there is no unique distance from the failure surface to the origin of the reduced variates. The tangent hyperplane to the failure surface at  $(x_1^*, x_2^*, \dots, x_n^*)$  may be used to approximate the actual failure surface, Fig. 2.6. Depending on whether the exact nonlinear failure surface

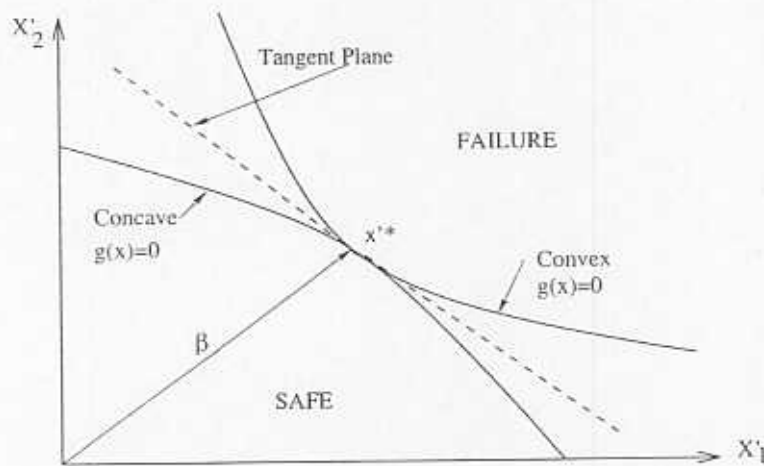


Figure 2.6: Non Linear Limit State

is convex or concave toward the origin, this approximation will be on the safe or unsafe side respectively.

The tangent hyperplane at  $x^* = (x_1^*, x_2^*, \dots, x_n^*)$  is

$$\sum_i^n (X'_i - x_i^*) \left( \frac{\partial g}{\partial X'_i} \right)_* = 0 \quad (2.64)$$

where the partial derivatives  $(\partial g / \partial X'_i)_*$  are evaluated at  $x^*$ . Thus, the distance from the “minimum” tangent hyperplane, Eq. 2.64 to the origin of the reduced variates is the appropriate reliability index.

Since the performance function is nonlinear, the pertinent point of tangency on the failure

surface is not known *a priori*. This point may be determined through the Lagrange multiplier method. Recalling that

$$x_i^* = \sigma_{X_i} x_i'^* + \mu_{X_i} = \mu_{X_i} - \alpha_i^* \sigma_{X_i} \beta \quad (2.65)$$

and the solution of the limit state equation

$$g(x_1^*, x_2^*, \dots, x_n^*) = 0 \quad (2.66)$$

yields the value of  $\beta$

Thus, the numerical algorithm to solve for  $\beta$  is as follows:

1. Assume initial values of  $x_i^*$ ;  $i = 1, 2, \dots, n$  and obtain

$$x_i'^* = \frac{x_i^* - \mu_{X_i}}{\sigma_{X_i}} \quad (2.67)$$

2. Evaluate  $(\partial g / \partial X_i)_{\bullet}$  and  $\alpha_i^*$  at  $x_i^*$ .
3. Form  $x_i^* = \mu_{X_i} - \alpha_i^* \sigma_{X_i} \beta$ .
4. Substitute above  $x_i^*$  in  $g(x_1^*, x_2^*, \dots, x_n^*) = 0$  and solve for  $\beta$ .
5. Using the  $\beta$  obtained in step 4, reevaluate  $x_i'^* = -\alpha_i \beta$ .
6. Repeat steps 2 through 5 until convergence is obtained. After obtaining the reliability index, the probability of failure can be calculated by

$$p_f = 1 - \Phi(\beta) \quad (2.68)$$

A flowchart illustrating this method is of determining the reliability index of a problem in terms of uncorrelated normal variables is shown in Fig. 2.7.

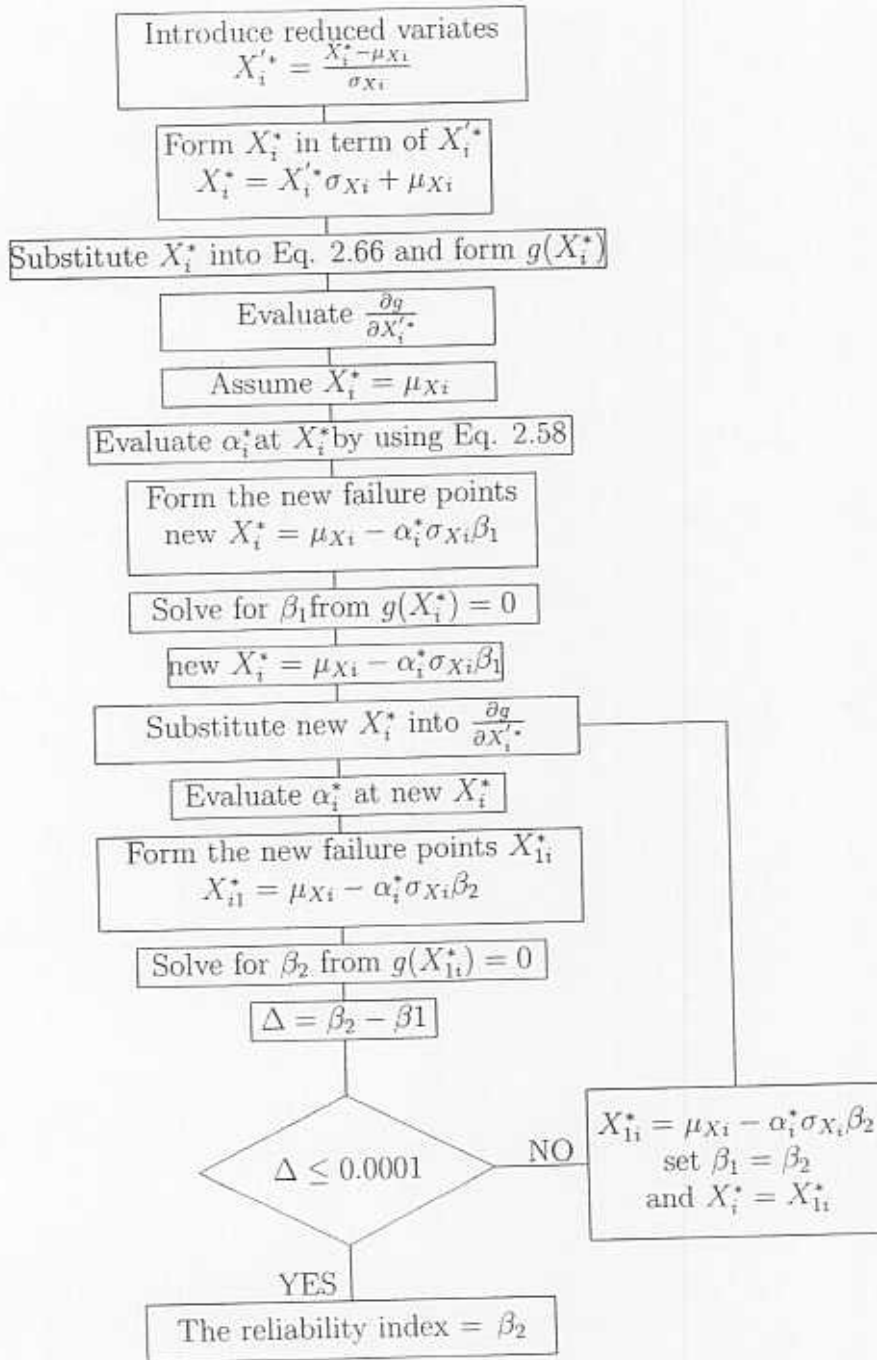


Figure 2.7: Determination of the Reliability Index in Terms of Uncorrelated Normal Variables

### 2.3.2 Equivalent normal distribution

If the probability distribution of random variable is not normal, we can adopt a method introduced in Ang and Tang (1984) to transform the non-normal distribution to the equivalent normal distribution. The basic concept behind this theory is that the cumulative probability and the probability density of the non-normal distribution has to be equal to the cumulative probability and probability density of the equivalent normal distribution at the appropriate point  $x_i^*$  at the failure point.

The result after transformation are the mean and standard deviation of the equivalent normal distribution. The procedure is as follows:

1. Equate the cumulative distribution of non-normal and equivalent normal distribution at the failure point  $x_i^*$ , we obtain

$$\Phi\left(\frac{x_i^* - \mu_{X_i}^N}{\sigma_{X_i}^N}\right) = F_{X_i}(x_i^*) \quad (2.69)$$

where  $\mu_{X_i}^N, \sigma_{X_i}^N$  are the mean and standard deviation of the equivalent normal distribution;  $F_{X_i}(x_i^*)$  is the original cumulative distribution function of  $X_i$  evaluated at  $x_i$ ;  $\Phi(-)$  is the cumulative distribution function of standard normal distribution. Hence, from 4.32, we obtain

$$\mu_{X_i}^N = x_i - \sigma_{X_i}^N \Phi^{-1}[F_{X_i}(x_i^*)] \quad (2.70)$$

As a result, we obtain  $\mu_{X_i}^N$  in term of  $\sigma_{X_i}^N$ .

2. Equate the PDF of non-normal and equivalent normal distribution at the failure point  $x_i^*$ , we obtain

$$\frac{1}{\sigma_{X_i}^N} \phi\left(\frac{x_i^* - \mu_{X_i}^N}{\sigma_{X_i}^N}\right) = f_{X_i}(x_i^*) \quad (2.71)$$

where  $\phi(-)$  is the probability density function of the standard normal distribution.

From this equation, we obtain

$$\sigma_{X_i}^N = \frac{\phi(\Phi^{-1}[F_{X_i}(x_i^*)])}{f_{X_i}(x_i^*)} \quad (2.72)$$



finally, we substitute the value of  $\sigma_{X_i}^N$  into Eq. 2.70

Using this transformation method, one can obtain the equivalent normal distribution at a point for non-normal ones. This method is summarized in Fig. 2.8

## 2.4 Conclusions

This chapter has presented in some details both the concept of reliability index and the approach to determine numerically (Monte-Carlo simulations), or approximated through analytical methods. In both cases, normal and non-normal distributions are considered. In the next chapter, the computer implementation in this research will be discussed.

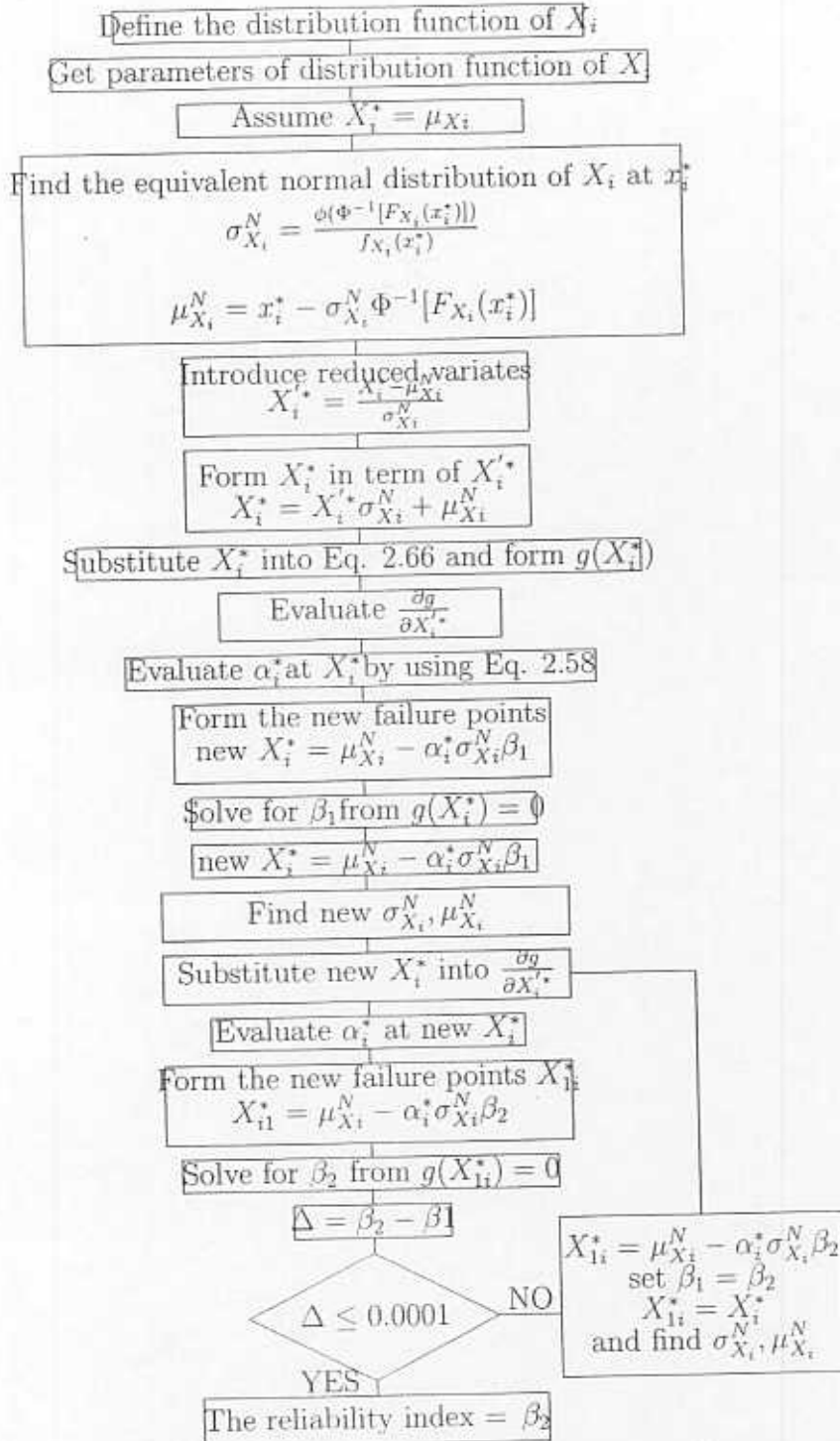


Figure 2.8: Flow Chart of the reliability evaluation for non-normal uncorelated variables

# Chapter 3

## Implementation

### 3.1 Overview

In this research, a Windows application names "RAP" (Reliability Analysis Program) is developed by using Microsoft Visual C++ .net 2003 for evaluating the reliability of the structure which can be indicated in term of reliability index and the probability of failure. It runs under Windows 2000 and Windows XP. The minimum requirement of the hardware are 133MHZ pentium processor, 64MB of RAM and 1 MB of Space on hard drive.

The program enables the user of the program to deal with stochastic quantities, so-called variables, in some given mathematical expression. In view of one of the applications of the program, this expression is called a limit state function. The program lends itself to reliability analysis, but may be used in a much wider context when evaluating the influence of variables for problems encountered in other fields of engineering practice.

### 3.2 Concept of the program

RAP is designed to have one main dialog for the user to be able to input, run the program and look at the result of the computation. When the program runs, the window of the program will look like in Fig.3.1. First, the variables have to be described by choosing among a set of several distribution types. After putting all information needed in each

variable, the user then need to click the "Add variable" button to put the random variable into the list of variables. The user can also removed the unwanted random variable from the list by selecting the random variable from the list and then clicking on the "Remove variable" button. After all random variables have been defined the user has to define the limit state or the performance function of the problem as a function of the random variables which already are defined. To define a limit state function, the following operators and functions may be used:

**RAP - Reliability Analysis Program**

Random Variable Definitions

Number of Variables:

Variable Name:  Distribution Type:

Name	Distribution Type	P1	P2	P3

Performance Function

performance =

Analysis Method

☒ Monte Carlo Simulation ☐ FORM Sample size:

Reliability Analysis Results

Number of samples:   
 Probability of failure:   
 Probability of survival:   
 Reliability Index:

View Output

Figure 3.1: RAP program

1. +, -, \*, /

2. Exponents with a preceding circumflex, for example,  $x^2$ ,  $x^2(a+b)$  or  $x^{-2}$ , respectively.
3. `sqrt(...)` and `sqr(...)` as alternatives for writing  $x^{(1/2)}$  and  $x^2$
4. Transcendental functions: `cos(...)`, `sin(...)`, `tan(...)`, `arccos(...)`, `arcsin(...)`, `arctan(...)`, `cosh(...)`, `sinh(...)`, `tanh(...)`, `arcosh(...)`, `arsinh(...)`, `artanh(...)`, `exp(...)`, `ln(...)`

Then select the method used for performing reliability analysis of the problem. There are 2 methods available in this program, the Monte-Carlo Simulation method and the First Order Reliability Method. If the user selects, the Monte-Carlo Simulation method, the user must define the number of sample used to perform the analysis. To begin the computation, the user has to click on the "Compute" button and then the computation starts until it finishes. After the computation finishes, the output is shown in the main dialog. If the user wants to look at the details of the computation, the user can click on the "Detail" button, the program will call "notepad.exe" to run notepad program and read the output file. Also if the user wants to look at the graph, the user can click the "Plot" button from the main dialog and the program will call "gnuplot.exe" to run gnuplot program to show the graph. The process of the computation part can be summarized as shown in Fig.3.2

### 3.3 Illustration

In this section, an illustration of evaluating reliability index of the selected problem by using RAP program is shown.

To use the RAP program for this problem, the steps are the following;

1. Run RAP.exe.
2. Define each random variable by putting its name into the "Variable name" box, selecting its distribution type, then filling the parameter need for the selected distribution and finally clicking on "Add variable" button. After finish defining the variables, the dialog will look like in Fig.3.3.

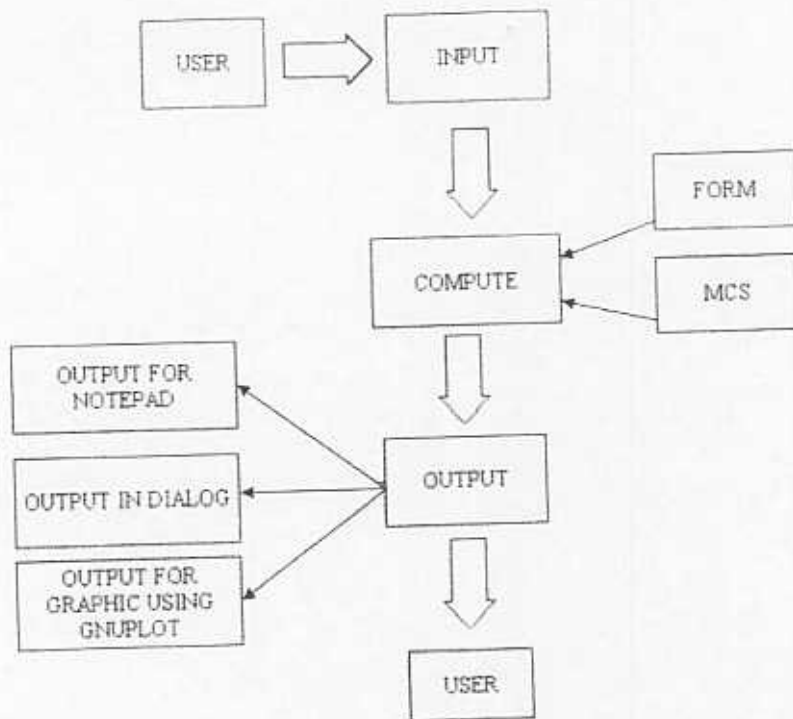


Figure 3.2: The process of computation part of RAP program

3. Define Performance function into the "Performance function" edit box
4. Select the method used for analysis. In this case, we select the Monte-Carlo Simulation method to perform the computation, So we need to select the number of the samples to consider. In this case, select 10,000 samples. Then the dialog will look like in Fig.3.4.
5. Click "Compute" button, the computation begins.
6. After the computation finishes, the results are shown from the dialog as shown in Fig.3.5.
7. Click "Plot" button to look at graph. In this case, we select the graph the PDF of each random variable. we obtain as shown in Fig.3.6.
8. To exit from the program, click the "Exit" button.

**RAP - Reliability Analysis Program**

Random Variable Definitions

Number of Variables:

Variable Name    Distribution Type

Name	Distribution Type	P1	P2
M	Normal	1000.000	200.000
Z	Normal	50.000	2.500
Y	Normal	40.000	5.000

Performance Function

performance =

Analysis Method

☒ MCS    Sample size

☐ FORM   

Reliability Analysis Results

Number of samples

Probability of failure

Probability of survival

Reliability Index

View Output

Show data

Plot

Figure 3.3: Process of adding variable

**RAP - Reliability Analysis Program**

Random Variable Definitions

Number of Variables:

Variable Name:  Distribution Type:

Name	Distribution Type	P1	P2
M	Normal	1000.000	200.000
Z	Normal	50.000	2.500
Y	Normal	40.000	5.000

Performance Function

performance =  $Y^2 Z M$

Analysis Method

☒ MCS Sample size:

☐ FORM

Reliability Analysis Results

Number of samples:

Probability of failure:

Probability of survival:

Reliability Index:

View Output

Show data:

Plot:

Figure 3.4: Process of selecting method for analysis



**RAP - Reliability Analysis Program**

Random Variable Definitions:

Number of Variables:

Variable Name:  Distribution Type:

Name	Distribution Type	P1	P2
M	Normal	1000.000	200.000
Z	Normal	50.000	2.500
Y	Normal	40.000	5.000

Performance Function:

performance =

Analysis Method:

☒ MCS Sample size:

☐ FORM

Reliability Analysis Results:

Number of samples:

Probability of failure:

Probability of survival:

Reliability Index:

View Output:

Show data:

Plot:

Figure 3.5: Window after completion of analyses

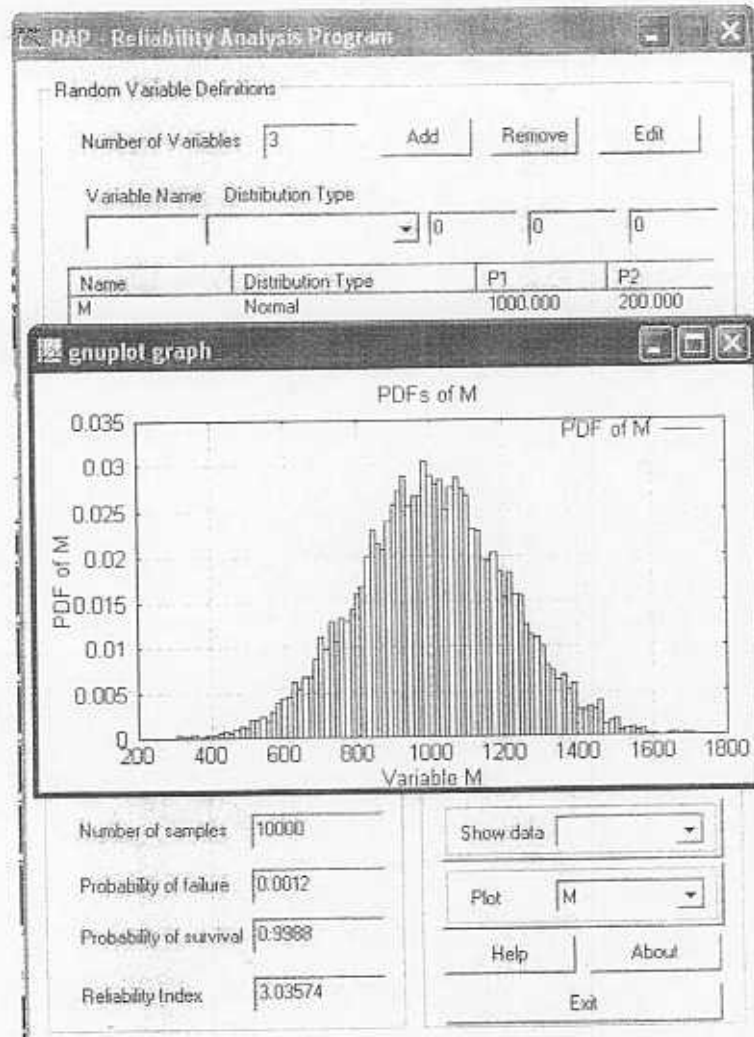


Figure 3.6: Example of graphical display of results

From the illustration, it can be seen that the RAP program has the graphical user interface that is very simple and easy to use. Also, the program is not restricted to a particular problem, but allows users to define functions on input.

In the next chapter, the program will be used to perform the reliability analysis of some examples. The results from the analyses will then be validated with the results from the reference and another software.

# Chapter 4

## Numerical Examples

### 4.1 Overview

In this chapter, some numerical examples taken from Ang and Tang (1984) are shown. Then RAP and VaP program (as shown in Figs. 4.1 and 4.2) will be used to perform the reliability analysis of the given examples. Both First Order Reliability Method and Monte-Carlo simulation method are used in each program to evaluate the reliability indices and also the probability of failures. Finally, the results from all computations are compared.

The numerical examples are divided into two parts which are: 1) all random variables are assumed to be normal random variates and 2) the combination of different kind of random variables.

The examples are as follows.

### 4.2 All random variables are assumed to be normal random variates

In this section, the comparison is considered when all random variables are assumed to be normal random variates. The example is as follows. The fully plastic flexural capacity of a steel beam section may be given as  $YZ$ , where  $Y$  is the yield strength of steel, and  $Z$  the plastic section modulus of the section.

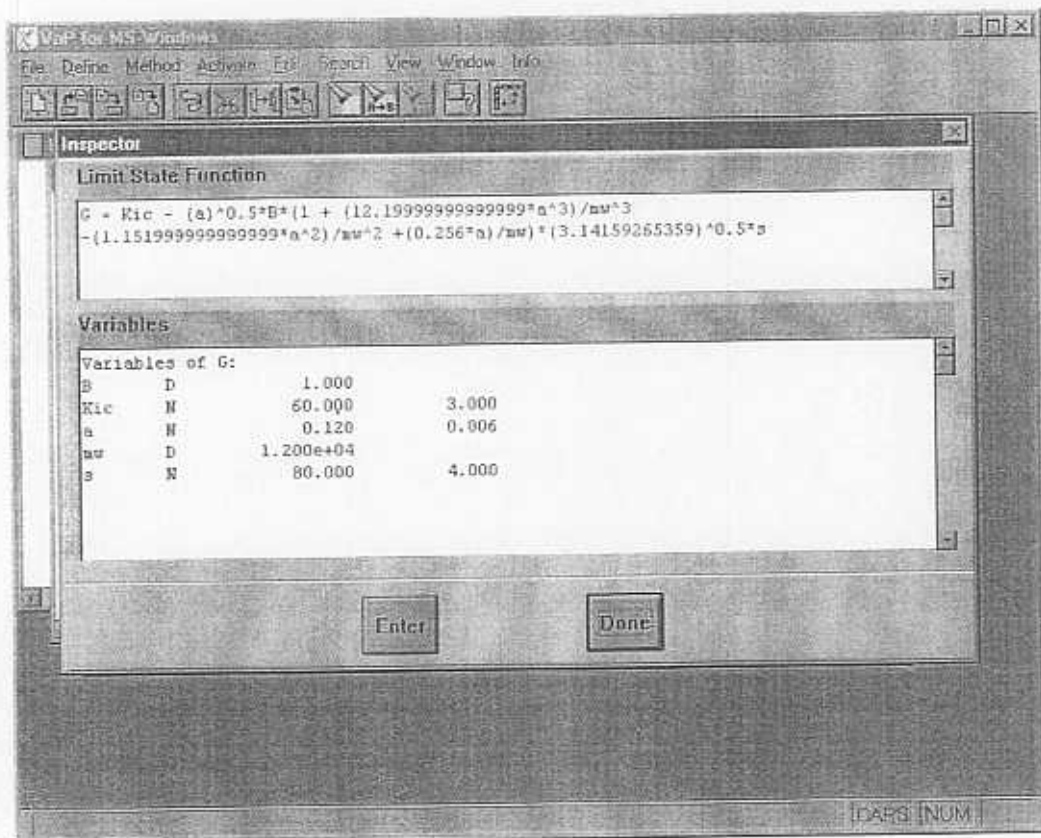


Figure 4.1: Input dialog of the VaP program

Then, if the applied bending moment at the pertinent section is  $M$ , the performance function may be defined as

$$g(X) = YZ - M \quad (4.1)$$

We assume that the variables are uncorrelated, and consider a beam with  $\bar{Y}=40$  ksi and  $\bar{Z}=50$  in<sup>3</sup> subjected to  $\bar{M}=1,000$  in-kip; the corresponding coefficients of variation are

$$\Omega_Y = 0.125 \quad \Omega_Z = 0.05 \quad \Omega_M = 0.2 \quad (4.2)$$

We seek to determine the reliability of the beam.

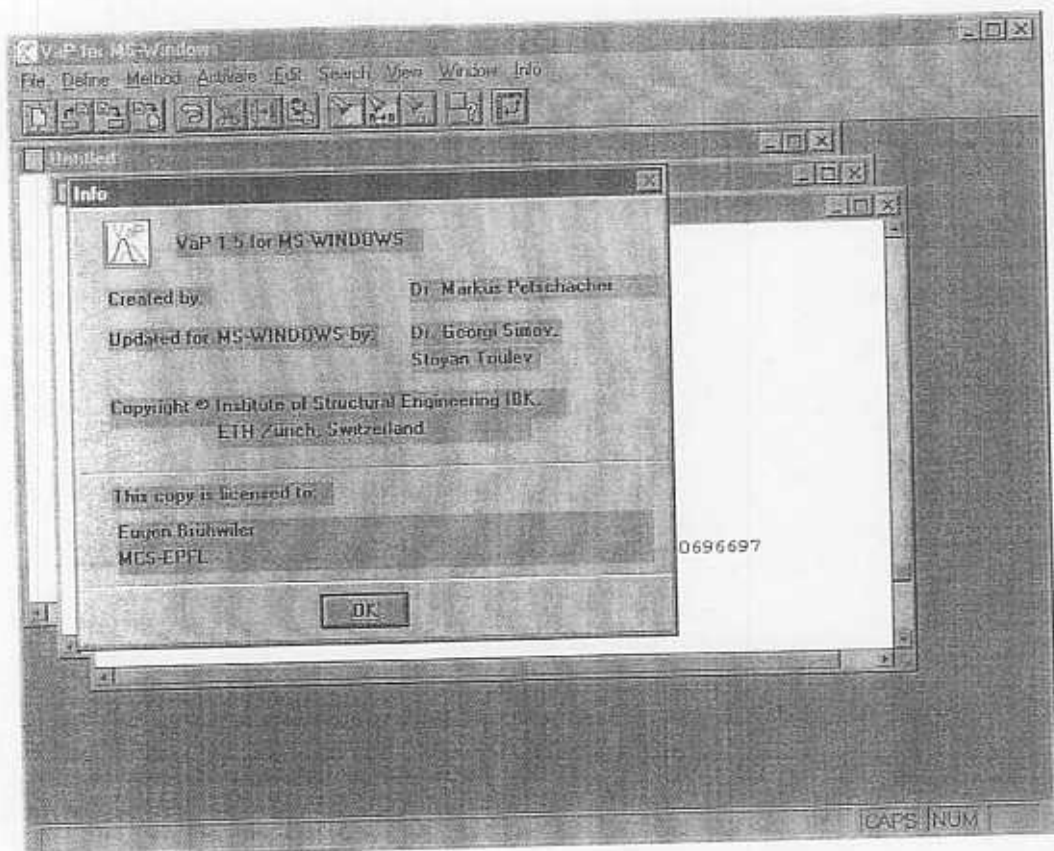


Figure 4.2: The VaP program

The corresponding standard deviations are:

$$\sigma_Y = 0.125 \times 40 = 5.0 \text{ ksi} \quad (4.3)$$

$$\sigma_Z = 0.05 \times 50 = 2.5 \text{ in}^3 \quad (4.4)$$

$$\sigma_M = 0.2 \times 1,000 = 200 \text{ in} - \text{kips} \quad (4.5)$$

In this particular case, the derivatives are

$$\left( \frac{\partial g}{\partial Y'} \right) = \sigma_Y Z \quad (4.6)$$

$$\left( \frac{\partial g}{\partial Z'} \right) = \sigma_Z Y \quad (4.7)$$

$$\left( \frac{\partial g}{\partial M'} \right) = -\sigma_M \quad (4.8)$$

For the first iteration, we assume  $Y^* = \bar{Y} = 40$  ksi;  $Z^* = \bar{Z} = 50$  in<sup>3</sup>; and  $M^* = \bar{M} = 1,000$  in-kip. Then

$$\left(\frac{\partial g}{\partial Y'}\right)_* = 5.0 \times 50 = 250 \quad (4.9)$$

$$\left(\frac{\partial g}{\partial Z'}\right)_* = 2.5 \times 40 = 100 \quad (4.10)$$

$$\left(\frac{\partial g}{\partial M'}\right)_* = -200 \quad (4.11)$$

The direction cosines therefore are:

$$\alpha_{Y'}^* = \frac{250}{\sqrt{(250)^2 + (100)^2 + (200)^2}} = \frac{250}{335.41} = 0.745 \quad (4.12)$$

$$\alpha_{Z'}^* = \frac{100}{335.41} = 0.298 \quad (4.13)$$

$$\alpha_{M'}^* = \frac{-200}{335.41} = 0.596 \quad (4.14)$$

Hence, the components of the failure point are

$$y^* = 40 - 0.745 \times 5.0\beta = 40 - 3.725\beta \quad (4.15)$$

$$z^* = 50 - 0.298 \times 2.5\beta = 50 - 0.745\beta \quad (4.16)$$

$$m^* = 1,000 + 0.596 \times 200\beta = 1,000 + 119.2\beta \quad (4.17)$$

Substituting these into the limit-state equation,  $y^* z^* - m^* = 0$ , yields the following quadratic equation

$$2.775\beta^2 - 335.25\beta + 1,000 = 0 \quad (4.18)$$

from which we obtain the solution  $\beta = 3.06$

The revised failure point then becomes

$$y^* = 40 - 3.725 \times 3.06 = 28.60 \quad (4.19)$$

$$z^* = 50 - 0.745 \times 3.06 = 47.72 \quad (4.20)$$

$$m^* = 1,000 + 119.2 \times 3.06 = 1,364.75 \quad (4.21)$$

Repeating the procedure for subsequent iterations, the result are summarized in Table 4.1.

Therefore, the probability of failure is

Table 4.1: Summary of Iterations for the Reliability index of a Plastic Beam

Iteration No.	Variable	Assumed Failure Point	$\left(\frac{\partial g}{\partial X_i}\right)_*$	$\alpha_{X_i}^*$	New $x_i^*$
1	Y	40	250	0.745	$40-3.725\beta$
	Z	50	100	0.298	$50-0.745\beta$
	M	1,000	-200	-0.596	$1,000 +119.20\beta$
$\beta = 3.06$					
2	Y	28.60	238.60	0.747	$40-3.735\beta$
	Z	47.72	71.50	0.224	$50-0.560\beta$
	M	1,364.75	-200.00	-0.626	$1,000 +125.20\beta$
$\beta = 3.05$					
3	Y	28.61	238.60	0.747	$40-3.735\beta$
	Z	48.29	71.50	0.224	$50-0.560\beta$
	M	1,381.86	-200.00	-0.626	$1,000 +125.20\beta$
$\beta = 3.05$					

$$p_f = 1 - \Phi(3.05) = 0.0014$$

Next, RAP program and VaP program are used to perform the reliability analysis for the above problem. After the computations, the results of the reliability index and the probability of failure which are obtained from RAP program and VaP program can be shown in the table 4.4.



Table 4.2: Comparison of reliability index and probability of failure for the example case where all random variables are assumed to be normal random variates using RAP and VaP

Method	Program	Reliability Index	Probability of failure
FORM	RAP	3.04908	0.001148
	VaP	3.05	0.00115
Monte-Carlo (100,000 samples)	RAP 3.30825		0.00119
	VaP	-	0.00129

### 4.3 The combination of different kind of random variables

As an illustrative example of the procedure, we consider the same example as shown in Section 4.2 which is the reliability of the steel beam. Recall that the fully plastic flexural capacity of a steel beam section can be given by  $YZ$  where  $Y$  is the yield strength of steel, and  $Z$  the section modulus of the section. If the applied bending moment at the pertinent section is  $M$ . Assume that  $Y$  is Lognormal,  $Z$  Lognormal, and  $M$  to be Type I asymptotic extreme. The performance function is given by

$$g(X) = YZ - M \quad (4.22)$$

Assume that the variates are uncorelated. Using the same design variables as in the example of Section 4.2, namely,  $\bar{Y} = 40$  ksi,  $\Omega_Y = 0.125$ ;  $\bar{Z} = 50$  in<sup>3</sup>,  $\Omega_Z = 0.05$ ;  $\bar{M} = 100$  in-kips,  $\Omega_M = 0.20$ , we obtain the distribution parameters as follows. For  $Y$  and  $Z$ , the parameters of the lognormal distributions are

$$\zeta_Y \simeq \Omega_Y = 0.125; \quad \lambda_Y = \ln 40 - \frac{1}{2}(0.125)^2 = 3.681 \quad (4.23)$$

$$\zeta_Z \simeq \Omega_Z = 0.05; \quad \lambda_Z = \ln 50 - \frac{1}{2}(0.05)^2 = 3.911 \quad (4.24)$$

The corresponding parameters of the Type I asymptotic distribution of  $M$  are

$$\alpha = \frac{\pi}{\sqrt{6}} \frac{1}{\sigma_M} = \frac{\pi}{\sqrt{6} \cdot 200} = 0.006413 \quad (4.25)$$

$$u = \bar{M} - \frac{0.577}{\alpha} = 1000 - \frac{0.577}{0.006413} = 910.02 \quad (4.26)$$

The partial derivatives with respect to the reduced variates are

$$\left( \frac{\partial g}{\partial Y'} \right) = \sigma_Y Z; \quad \left( \frac{\partial g}{\partial Z'} \right) = \sigma_Z Y; \quad \left( \frac{\partial g}{\partial M'} \right) = \sigma_M \quad (4.27)$$

For the lognormal distribution, the PDF and CDF of random variable  $Y$  are

$$F_Y(y) = \Phi \left( \frac{\ln y - \lambda_Y}{\zeta_Y} \right) \quad (4.28)$$

$$f_Y(y) = \frac{1}{y \zeta_Y} \phi \left( \frac{\ln y - \lambda_Y}{\zeta_Y} \right) \quad (4.29)$$

Then, Eqs. 2.70 and 2.72, respectively, yield

$$\sigma_Y^N = \frac{1}{f_Y(y^*)} \phi \left( \Phi^{-1} \left[ \Phi \left( \frac{\ln y^* - \lambda_Y}{\zeta_Y} \right) \right] \right) \quad (4.30)$$

$$= \frac{1}{f_Y(y)} \phi \left( \frac{\ln y - \lambda_Y}{\zeta_Y} \right) \quad (4.31)$$

$$= y^* \zeta_Y \quad (4.32)$$

and

$$\mu_Y^N = y^* - \sigma_Y^N \Phi^{-1} \left[ \Phi \left( \frac{\ln y^* - \lambda_Y}{\zeta_Y} \right) \right] \quad (4.33)$$

$$= y^* - y^* \zeta_Y \left( \frac{\ln y^* - \lambda_Y}{\zeta_Y} \right) \quad (4.34)$$

$$= y^* (1 - \ln y^* + \lambda_Y) \quad (4.35)$$

From Eqs. 4.32 and 4.35, we obtain

$$\sigma_Z^N = z^* \zeta_Z \quad (4.36)$$

and

$$\mu_Z^N = z^* (1 - \ln z^* + \lambda_Z) \quad (4.37)$$

whereas for the Type I asymptotic distribution of  $M$ ,

$$F_M(m) = \exp[-e^{-\alpha(m-u)}] \quad (4.38)$$

$$f_M(m) = \alpha \exp[-\alpha(m-u) - e^{-\alpha(m-u)}] \quad (4.39)$$

and

$$\mu_M^N = m^* - \sigma_M^N \Phi^{-1}[F_M(m^*)] \quad (4.40)$$

$$\sigma_M^N = \frac{\phi(\Phi^{-1}[F_M(m^*)])}{f_M(m^*)} \quad (4.41)$$

Using these relationships and assuming for the first iteration.

$$y^* = \bar{Y} = 40; \quad z^* = \bar{Z} = 50; \quad m^* = \bar{M} = 1000 \quad (4.42)$$

we obtain

$$\sigma_Y^N = 40 \times 0.125 = 5.0 \quad (4.43)$$

$$\mu_Y^N = 40(1 - \ln 40 + 3.681) = 39.69 \quad (4.44)$$

$$\sigma_Z^N = 50 \times 0.05 = 2.5 \quad (4.45)$$

$$\mu_Z^N = 50(1 - \ln 50 + 3.911) = 49.95 \quad (4.46)$$

$$F_M(m^*) = \exp[e^{-0.006413(100-910.02)-0.5616}] = e^{-0.5616} = 0.5703 \quad (4.47)$$

$$\begin{aligned} f_M(m^*) &= 0.006413 \exp[-0.006413(100 - 910.02) - 0.5616] \\ &= 0.002054 \end{aligned} \quad (4.48)$$

$$\sigma_M^N = \frac{\phi[\Phi^{-1}(0.5703)]}{0.002054} = 191.14 \quad (4.49)$$

$$\mu_M^N = 1000 - 191.14\Phi^{-1}(0.5703) = 965.78 \quad (4.50)$$

The partial derivatives are

$$\left(\frac{\partial g}{\partial Y'}\right)_* = \sigma_Y^N z^* = 50 \times 50 = 250 \quad (4.51)$$

$$\left(\frac{\partial g}{\partial Z'}\right)_* = \sigma_Z^N y^* = 40 \times 2.5 = 100 \quad (4.52)$$

$$\left(\frac{\partial g}{\partial M'}\right)_* = -\sigma_M^N = -191.14 \quad (4.53)$$

From which the direction cosine are

$$\alpha_Y^* = \frac{250}{\sqrt{(250)^2 + (100)^2 + (191.14)^2}} = \frac{250}{330.2} = 0.757 \quad (4.54)$$

$$\alpha_Z^* = \frac{100}{330.2} = 0.303 \quad (4.55)$$

$$\alpha_M^* = \frac{-191.14}{330.2} = -0.580 \quad (4.56)$$

therefore,

$$y^* = \mu_Y^N - \alpha_Y^* \beta \sigma_Y^N = 39.69 - 0.757 \times 5.0\beta = 39.69 - 3.786\beta \quad (4.57)$$

$$z^* = \mu_Z^N - \alpha_Z^* \beta \sigma_Z^N = 49.85 - 0.303 \times 2.5\beta = 49.85 - 0.757\beta \quad (4.58)$$

$$\begin{aligned} m^* &= \mu_M^N - \alpha_M^* \beta \sigma_M^N = 965.87 + 0.580 \times 191.14\beta \\ &= 965.78 + 110.78\beta \end{aligned} \quad (4.59)$$

$$(4.60)$$

The limit-state equation then becomes

$$2.86\beta^2 - 329.67\beta + 1011.86 = 0 \quad (4.61)$$

The pertinent solution is

$$\beta = 3.156 \quad (4.62)$$

Thus, the new failure point is:

$$y^* = 27.69 \quad (4.63)$$

$$z^* = 47.50 \quad (4.64)$$

$$m^* = 1315.40 \quad (4.65)$$

The results of the iterations may be summarized as shown in Table 4.3. Therefore, the probability of failure is

$$p_f = 1 - \Phi(2.745) = 0.00302 \quad (4.66)$$

Next, RAP program and VaP program are used to perform the reliability analysis for

Table 4.3: Summary of Iterations for the Reliability index of a Plastic Beam

Iteration No.	Variable	Assumed Failure Point	$\sigma_{X_i}^N$	$\mu_{X_i}^N$	$\left(\frac{\partial g}{\partial X_i}\right)_*$	$\alpha_{X_i}^*$	New $x_i^*$
1	Y	40	5.0	39.69	250	0.757	$39.69-3.786\beta$
	Z	50	2.5	49.95	100	0.303	$49.94-0.757\beta$
	M	1,000	191.14	965.78	-191.14	-0.580	$965.78+110.78\beta$
$\beta = 3.156$							
2	Y	27.69	3.461	37.63	164.4	0.462	$37.63-1.599\beta$
	Z	47.50	2.375	49.84	65.76	0.185	$49.84-0.439\beta$
	M	1315.40	308.8	863.3	-308.8	-0.867	$863.3+267.73\beta$
$\beta = 2.796$							
3	Y	33.16	4.145	39.08	201.5	0.432	$39.08-1.791\beta$
	Z	48.61	2.431	49.88	80.61	0.173	$49.88-0.421\beta$
	M	1611.9	412.3	667.8	-412.3	-0.885	$667.8+364.9\beta$
$\beta = 2.735$							
4	Y	34.18	4.273	39.29	208.22	0.432	$39.29-1.846\beta$
	Z	48.73	2.437	49.93	83.30	0.173	$49.93-0.422\beta$
	M	1665.8	426.3	634.2	-426.3	-0.885	$634.2+377.3\beta$
$\beta = 2.745$							

the above problem. After the computations, the results of the reliability index and the probability of failure which are obtained from RAP program and VaP program can be shown in the table 4.4.

Table 4.4: Comparison of reliability index and probability of failure for the example case where all random variables are assumed to be normal random variates using RAP and VaP

Method	Program	Reliability Index	Probability of failure
FORM	RAP	2.74056	0.00306681
	VaP	2.74	0.00307
Monte-Carlo (100,000 samples)	RAP	2.76575	0.00284
	VaP	-	0.00323

## 4.4 Conclusions

From the comparisons of the results obtained from the RAP and VaP programs to Ang and Tang (1984), the conclusions are as the follows. First, by using First Order Reliability Method, the results from Ang and Tang (1984), RAP program and VaP program are very closed in all of the consideration cases. Secondly, by using Monte-Carlo Simulation Method, the results from Ang and Tang (1984), RAP program and VaP program are very closed in all of the consideration cases. Finally, the results of the probability of failure from the two methods are a little bit different. It has been shown in Ang and Tang (1984) that the large samples of Monte-Carlo Simulation will give more correct probability of failure than the First Order Reliability Method in the case of non-linear performance function due to the linear approximation of the non-linear performance function of the later method.

Accordingly, the conclusion and the recommendation for future work of this study can be presented in the next chapter.

# Chapter 5

## Conclusions

RAP is a windows application for evaluating the reliability index and the probability of failure of the given performance function of the structure. The reliability methods used are First Order Reliability Method (FORM) and Monte-Carlo Simulation method (MCS). The results of the computation of the program are reliability index and probability of failure. The comparisons of the results from RAP with the ones from Ang and Tang (1984) and the ones from the VaP programs have been considered. The result from RAP program is almost the same as the ones from Ang and Tang (1984) and VaP programs in both methods in every considered problem. Finally, the comparison of the result of the reliability analysis from the two reliability methods has been considered. In the non-linear performance function of problem, the result from Monte-Carlo Simulation with a large number of samples will give more correct result than the one from First Order Reliability Method.

### 5.1 Recommendation for Future Work

Two recommendations for future work are as follows;

1. Implement the RAP program to evaluate the system reliability of the structure which has many failure modes inside it.
2. Implement the RAP program to evaluate the reliability of the general structure by using probabilistic finite element method.



# Chapter 5

## Conclusions

RAP is a windows application for evaluating the reliability index and the probability of failure of the given performance function of the structure. The reliability methods used are First Order Reliability Method (FORM) and Monte-Carlo Simulation method (MCS). The results of the computation of the program are reliability index and probability of failure. The comparisons of the results from RAP with the ones from Ang and Tang (1984) and the ones from the VaP programs have been considered. The result from RAP program is almost the same as the ones from Ang and Tang (1984) and VaP programs in both methods in every considered problem. Finally, the comparison of the result of the reliability analysis from the two reliability methods has been considered. In the non-linear performance function of problem, the result from Monte-Carlo Simulation with a large number of samples will give more correct result than the one from First Order Reliability Method.

### 5.1 Recommendation for Future Work

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# Appendix A

## Table of Standard Normal Probability

Table A.1: Table of Standard Normal Probability

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
0.00	0.500000	0.20	0.579260	0.40	0.655422	0.60	0.725747
0.01	0.503989	0.21	0.583166	0.41	0.659097	0.61	0.729069
0.02	0.507978	0.22	0.587064	0.42	0.662757	0.62	0.732371
0.03	0.511966	0.23	0.590954	0.43	0.666402	0.63	0.735653
0.04	0.515954	0.24	0.594835	0.44	0.670032	0.64	0.738914
0.05	0.519939	0.25	0.598706	0.45	0.673645	0.65	0.742154
0.06	0.523922	0.26	0.602568	0.46	0.677242	0.66	0.745374
0.07	0.527904	0.27	0.606420	0.47	0.680823	0.67	0.748572
0.08	0.531882	0.28	0.610262	0.48	0.684387	0.68	0.751748
0.09	0.535857	0.29	0.614092	0.49	0.687933	0.69	0.754903
0.10	0.539828	0.30	0.617912	0.50	0.691463	0.70	0.758036
0.11	0.543796	0.31	0.621720	0.51	0.694975	0.71	0.761148
0.12	0.547759	0.32	0.625517	0.52	0.698468	0.72	0.764238
0.13	0.551717	0.33	0.629301	0.53	0.701944	0.73	0.767305
0.14	0.555671	0.34	0.633072	0.54	0.705401	0.74	0.770305
0.15	0.559618	0.35	0.636831	0.55	0.708804	0.75	0.773373
0.16	0.563560	0.36	0.640576	0.56	0.712260	0.76	0.776373
0.17	0.567494	0.37	0.644309	0.57	0.715661	0.77	0.779350
0.18	0.571423	0.38	0.648027	0.58	0.719043	0.78	0.782305
0.19	0.575345	0.39	0.651732	0.59	0.722405	0.79	0.785236
0.20	0.579260	0.40	0.655422	0.60	0.725747	0.80	0.788145

Table A.1 (continued)

$x$	$\Phi(x)$	$x$	$\Phi(x)$	$x$	$\Phi(x)$	$x$	$\Phi(x)$
0.80	0.788145	1.00	0.841345	1.20	0.884930	1.40	0.919243
0.81	0.791030	1.01	0.843752	1.21	0.886860	1.41	0.920730
0.82	0.793892	1.02	0.846136	1.22	0.888767	1.42	0.922196
0.83	0.796731	1.03	0.848495	1.23	0.890651	1.43	0.923641
0.84	0.799546	1.04	0.850830	1.24	0.892512	1.44	0.925066
0.85	0.802337	1.05	0.853141	1.25	0.894350	1.45	0.926471
0.86	0.805105	1.06	0.855428	1.26	0.896165	1.46	0.927855
0.87	0.807850	1.07	0.857690	1.27	0.897958	1.47	0.929219
0.88	0.810570	1.08	0.859929	1.28	0.899727	1.48	0.930563
0.89	0.813267	1.09	0.862143	1.29	0.901475	1.49	0.931888
0.90	0.815940	1.10	0.864334	1.30	0.903199	1.50	0.933193
0.91	0.818589	1.11	0.866500	1.31	0.904902	1.51	0.934478
0.92	0.821214	1.12	0.868643	1.32	0.906583	1.52	0.935744
0.93	0.823815	1.13	0.870762	1.33	0.908241	1.53	0.936992
0.94	0.826391	1.14	0.872857	1.34	0.909877	1.54	0.938220
0.95	0.828944	1.15	0.874928	1.35	0.911492	1.55	0.939429
0.96	0.831473	1.16	0.876976	1.36	0.913085	1.56	0.940620
0.97	0.833977	1.17	0.878999	1.37	0.914656	1.57	0.941792
0.98	0.836457	1.18	0.881000	1.38	0.916207	1.58	0.942947
0.99	0.838913	1.19	0.882977	1.39	0.917735	1.59	0.944083
1.00	0.841345	1.20	0.884930	1.40	0.919243	1.60	0.945201

Table A.1 (continued)

$x$	$\Phi(x)$	$x$	$\Phi(x)$	$x$	$\Phi(x)$	$x$	$\Phi(x)$
1.60	0.945201	1.80	0.964070	2.00	0.977250	2.20	0.986097
1.61	0.946301	1.81	0.964852	2.01	0.977784	2.21	0.986447
1.62	0.947384	1.82	0.965621	2.02	0.978308	2.22	0.986791
1.63	0.948449	1.83	0.966375	2.03	0.978822	2.23	0.987126
1.64	0.949497	1.84	0.967116	2.04	0.979325	2.24	0.987455
1.65	0.950529	1.85	0.967843	2.05	0.979818	2.25	0.987776
1.66	0.951543	1.86	0.968557	2.06	0.980301	2.26	0.988089
1.67	0.952540	1.87	0.969258	2.07	0.980774	2.27	0.988396
1.68	0.953521	1.88	0.969946	2.08	0.981237	2.28	0.988696
1.69	0.954486	1.89	0.970621	2.09	0.981691	2.29	0.988989
1.70	0.955435	1.90	0.971284	2.10	0.982136	2.30	0.989276
1.71	0.956367	1.91	0.971933	2.11	0.982571	2.31	0.989556
1.72	0.957284	1.92	0.972571	2.12	0.982997	2.32	0.989830
1.73	0.958185	1.93	0.973197	2.13	0.983414	2.33	0.990097
1.74	0.959071	1.94	0.973810	2.14	0.983823	2.34	0.990358
1.75	0.959941	1.95	0.974412	2.15	0.984223	2.35	0.990613
1.76	0.960796	1.96	0.975002	2.16	0.984614	2.36	0.990863
1.77	0.961636	1.97	0.975581	2.17	0.984997	2.37	0.991106
1.78	0.962462	1.98	0.976148	2.18	0.985371	2.38	0.991344
1.79	0.963273	1.99	0.976705	2.19	0.985738	2.39	0.991576
1.80	0.964070	2.00	0.977250	2.20	0.986097	2.40	0.991802

Table A.1 (continued)

$x$	$\Phi(x)$	$x$	$\Phi(x)$	$x$	$\Phi(x)$	$x$	$\Phi(x)$
2.40	0.991802	2.60	0.995339	2.80	0.997445	3.00	0.998650
2.41	0.992024	2.61	0.995473	2.81	0.997523	3.01	0.998694
2.42	0.992240	2.62	0.995604	2.82	0.997599	3.02	0.998736
2.43	0.992451	2.63	0.995731	2.83	0.997673	3.03	0.998777
2.44	0.992656	2.64	0.995855	2.84	0.997744	3.04	0.998817
2.45	0.992857	2.65	0.995975	2.85	0.997814	3.05	0.998856
2.46	0.993053	2.66	0.996093	2.86	0.997882	3.06	0.998893
2.47	0.993244	2.67	0.996207	2.87	0.997948	3.07	0.998930
2.48	0.993431	2.68	0.996319	2.88	0.998012	3.08	0.998965
2.49	0.993613	2.69	0.996427	2.89	0.998074	3.09	0.998999
2.50	0.993790	2.70	0.996533	2.90	0.998134	3.10	0.999032
2.51	0.993963	2.71	0.996636	2.91	0.998193	3.11	0.999065
2.52	0.994132	2.72	0.996736	2.92	0.998250	3.12	0.999096
2.53	0.994297	2.73	0.996833	2.93	0.998305	3.13	0.999126
2.54	0.994457	2.74	0.996928	2.94	0.998359	3.14	0.999155
2.55	0.994614	2.75	0.997020	2.95	0.998411	3.15	0.999184
2.56	0.994766	2.76	0.997110	2.96	0.998462	3.16	0.999211
2.57	0.994915	2.77	0.997197	2.97	0.998511	3.17	0.999238
2.58	0.995060	2.78	0.997282	2.98	0.998559	3.18	0.999264
2.59	0.995201	2.79	0.997365	2.99	0.998605	3.19	0.999289
2.60	0.995339	2.80	0.997445	3.00	0.998650	3.20	0.999313

Table A.1 (continued)

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
3.20	0.999313	3.40	0.999663	3.60	0.999841	3.80	0.999928
3.21	0.999336	3.41	0.999675	3.61	0.999847	3.81	0.999931
3.22	0.999359	3.42	0.999687	3.62	0.999853	3.82	0.999933
3.23	0.999381	3.43	0.999698	3.63	0.999858	3.83	0.999936
3.24	0.999402	3.44	0.999709	3.64	0.999864	3.84	0.999938
3.25	0.999423	3.45	0.999720	3.65	0.999869	3.85	0.999941
3.26	0.999443	3.46	0.999730	3.66	0.999874	3.86	0.999943
3.27	0.999462	3.47	0.999740	3.67	0.999879	3.87	0.999946
3.28	0.999481	3.48	0.999749	3.68	0.999883	3.88	0.999948
3.29	0.999499	3.49	0.999758	3.69	0.999888	3.89	0.999950
3.30	0.999516	3.50	0.999767	3.70	0.999892	3.90	0.999952
3.31	0.999533	3.51	0.999776	3.71	0.999896	3.91	0.999954
3.32	0.999550	3.52	0.999784	3.72	0.999900	3.92	0.999956
3.33	0.999566	3.53	0.999792	3.73	0.999904	3.93	0.999958
3.34	0.999581	3.54	0.999800	3.74	0.999908	3.94	0.999959
3.35	0.999596	3.55	0.999807	3.75	0.999912	3.95	0.999961
3.36	0.999610	3.56	0.999815	3.76	0.999915	3.96	0.999963
3.37	0.999624	3.57	0.999821	3.77	0.999918	3.97	0.999964
3.38	0.999637	3.58	0.999828	3.78	0.999922	3.98	0.999966
3.39	0.999650	3.59	0.999835	3.79	0.999925	3.99	0.999967
3.40	0.999663	3.60	0.999841	3.80	0.999928	4.00	0.999968



Table A.1 (continued)

x	$1 - \Phi(x)$	x	$1 - \Phi(x)$	x	$1 - \Phi(x)$
4	3.16712E-05	4.85	6.17307E-07	6.40	7.7688E-11
4.05	2.56088E-05	4.90	4.79183E-07	6.50	4.016E-11
4.10	2.06575E-05	4.95	3.71067E-07	6.60	2.0558E-11
4.15	1.66238E-05	5.00	2.86652E-07	6.70	1.0421E-11
4.20	1.33458E-05	5.10	1.69827E-07	6.80	5.231E-12
4.25	1.06885E-05	5.20	9.96443E-08	6.90	2.6E-12
4.30	8.53906E-06	5.30	5.79013E-08	7.00	1.28E-12
4.35	6.80688E-06	5.40	3.33204E-08	7.10	6.24E-13
4.40	5.41254E-06	5.50	1.89896E-08	7.20	3.01E-13
4.45	4.29351E-06	5.60	1.07176E-08	7.30	1.44E-13
4.50	3.39767E-06	5.70	5.99037E-09	7.40	6.8E-14
4.55	2.6823E-06	5.80	3.31575E-09	7.50	3.2E-14
4.60	2.11245E-06	5.90	1.81751E-09	7.60	1.5E-14
4.65	1.65968E-06	6.00	9.86588E-10	7.70	7E-15
4.70	1.30081E-06	6.10	5.30343E-10	7.80	3E-15
4.75	1.01708E-06	6.20	2.82316E-10	7.90	1.5E-15
4.80	7.93328E-07	6.30	1.48823E-10		
4.85	6.17307E-07	6.40	7.7688E-11		



## Appendix B

### Probability Distribution Library

Table B.1: Probability Distribution Library

Distribution	PDF	CDF	P1	P2	P3
Normal	$\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$	$\Phi\left[\frac{x-\mu}{\sigma}\right]$	$\mu$	$\sigma > 0$	—
Lognormal	$\frac{1}{\sqrt{2\pi}\zeta x} e^{-\frac{1}{2}\left(\frac{\ln x - \lambda}{\zeta}\right)^2}$	$\Phi\left[\frac{\ln x - \lambda}{\zeta}\right]$	$\lambda$	$\zeta > 0$	—
Shifted-Lognormal	$\frac{1}{\sqrt{2\pi}\zeta(x-x_0)} e^{-\frac{1}{2}\left(\frac{\ln(x-x_0) - \lambda}{\zeta}\right)^2}$	$\Phi\left[\frac{\ln(x-x_0) - \lambda}{\zeta}\right]$	$\lambda$	$\zeta > 0$	$x_0$
Shifted-Exponential	$\lambda e^{-\lambda(x-x_0)}$	$1 - e^{-\lambda(x-x_0)}$	$\lambda > 0$	$x_0$	—
Exponential	$\lambda e^{-\lambda x}$	$1 - e^{-\lambda x}$	$\lambda > 0$	—	—
Type I Asymptotic(largest)	$\alpha e^{-\alpha(x-x_n)} \exp[-e^{-\alpha(x-x_n)}]$	$\exp[-e^{-\alpha(x-x_n)}]$	$\alpha$	$u_n$	—
Type III (smallest)	$\frac{k}{w_1 - x_0} \left[ \frac{x - x_0}{w_1 - x_0} \right]^{k-1} e^{-\left(\frac{x-x_0}{w_1-x_0}\right)^k}$	$1 - e^{-\left(\frac{x-x_0}{w_1-x_0}\right)^k}$	$w_1$	$k > 0$	$x_0$

# Appendix C

## Vitae

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### EDUCATION

- Aug. 98 – Aug. 02    **University of Colorado**, Boulder, Colorado, USA.  
Ph.D., Structural Engineering    Aug. 2002  
Thesis: Numerical Simulation of Coupled Chemical-Mechanical Deterioration of Concrete.  
(Saouma, V.E.)
- Aug. 96 – Aug. 98    **University of Colorado**, Boulder, Colorado, USA.  
M.Sc., Civil Engineering;    Aug. 1998  
Thesis: Probabilistic Fracture Mechanics. (Saouma, V.E.)
- June 91 – Oct. 94    **Khon Kaen University (KKU)**, Khon Kaen, Thailand.  
B.Eng., Civil Engineering.    Oct. 1994

### RESEARCH EXPERIENCE

- Aug. 00 – Aug. 02    **Research Assistant**  
Department of Civil & Environmental Engineering, **University of Colorado**, Boulder,  
Colorado, USA

### WORK EXPERIENCE

- Apr.03 – present    **Assistant Professor**  
Department of Civil Engineering, UbonRatchathani  
University, Warinchamrab, UbonRatchathani 34190, Thailand
- Aug. 02 – Apr. 04    **Lecturer**  
Department of Civil Engineering, UbonRatchathani  
University, Warinchamrab, UbonRatchathani 34190, Thailand
- Dec. 93 – Aug. 96    **Lecturer**  
Department of Civil Engineering, UbonRatchathani  
University, Warinchamrab, UbonRatchathani 34190, Thailand

### COMPUTER SKILLS

- Operating systems: Window NT, Window 95/98, Window XP, UNIX
- Software: Visual C++, Sap2000, MS Word, MS Excel, MS Powerpoint

### SCHOLARSHIP/HONORS/AWARDS

- Oct. 04 – Sep. 05    **Research Grant:** Assessment of dynamic behaviour and loading capacity of railway bridges:  
Inspection and analytical technique
- Oct. 03 – Sep. 04    **Research Grant:** A study and development of windows base program of reliability analysis  
for assessing service life of cracked connections
- Aug. 96 – Aug. 02    Thai Government Scholarship for M.Sc./Ph.D. in Structural Engineering.

### SELECTED PUBLICATIONS/PRESENTATIONS

#### INTERNATIONAL JOURNAL

- 1 Puatatsananon, W. and Saouma, V., Nonlinear Coupling of Carbonation and Chloride Diffusion in Concrete,  
in-print ASCE J. of Materials Engineering, April 2005

#### NATIONAL JOURNAL

- 1 Kittisak Kuntiyawichai, **Wiwat Puatatsananon**, Griengsak Kaewkulchai and Suchart Limkatanyu, (2005), "A comparative study on dynamic response of different floor types subjected to walking load", *Submitted to KKU Engineering Journal*.
- 2 Kittisak Kuntiyawichai, **Wiwat Puatatsananon** and Suchart Limkatanyu, (2005), "A study and development of windows based program of reliability analysis for assessing service life of crack connections ", *Submitted to Songklanakarin J. Sci. Technol.*

#### INTERNATIONAL CONFERENCE PAPER

- 1 **W. Puatatsananon**, K. Kuntiyawichai, (2003), "Service life assessment of cruciform connection using probabilistic approach" The Fourth Regional Symposium on Infrastructure Development in Civil Engineering , RSID4, Thailand

#### NATIONAL CONFERENCE PAPERS

- 1 **Puatatsananon, W.**, Kuntiyawichai, K., Kaewkulchai, G. and Limkatanyu, S. (2005) "Effects of statistical variability of applied stress range and size of initial cracks in fatigue life of cruciform connection." The 10th National Convention on Civil Engineering 2003 at Chonburi, Petchaburi, pp. STR33.
- 2 Kaewkulchai, G., Bhokha, S., **Puatatsananon, W.**, and Kuntiyawichai, K. (2005) "Analysis for progressive collapse of building frames." The 10th National Convention on Civil Engineering 2003 at Chonburi, Petchaburi, pp. STR132.
- 3 Kaewkulchai, G., **Puatatsananon, W.**, Kuntiyawichai, K. and Limkatanyu, S. (2004) "Progressive Collapse of Building Frames." The 3rd PSU-Engineering Conference, PEC3, Prince of Songkla University, Songkla, Thailand, December 8-9, 2004.
- 4 Limkatanyu, S., **Puatatsananon, W.**, Kuntiyawichai, K. and Kaewkulchai, G. (2004) "Seismic Analysis of Reinforced Concrete Frames Including Reinforcement Slippage Effects." The 3rd PSU-Engineering Conference, PEC3, Prince of Songkla University, Songkla, Thailand, December 8-9, 2004.
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- 6 Kuntiyawichai, K., **Puatatsananon, W.** and Kaewkulchai, G. (2004). "Dynamic Behaviour of Through Truss Bridge (TT) under Passing Train." The 8th Annual National Symposium on Computational Science and Engineering, ANSCSE8, Suranaree University of Technology, Nakhonratchasima, Thailand, July 21-23, 2004.
- 7 S. Nilrat, **W. Puatatsananon**, K. Kuntiyawichai, 2004, "Vibration behaviour of long-span flat concrete floor subjected to human walking", The 9th National Convention on Civil Engineering 2003 at Cha-um, Petchaburi, pp. STR78-83.

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### EDUCATION

- Jan. 99 – Aug. 01     **The University of Manchester Institute of Science and Technology (UMIST),**  
Manchester, United Kingdom.  
Ph.D., Structural Engineering     Aug. 2001  
Thesis: Assessment of Fracture in Structures subjected to Dynamic Loading.  
(F.M. Burdekin)
- Sep. 97 – Sep. 98     **The University of Manchester Institute of Science and Technology (UMIST),**  
Manchester, United Kingdom.  
M.Sc., Structural Engineering;     Sep. 1998  
Thesis: Strength of Iron Column. (T. Swailes)
- June 92 – Oct. 95     **Khon Kaen University (KKU), Khon Kaen, Thailand.**  
B.Eng., Civil Engineering,     Oct. 1995  
Project: Prediction of pile loading capacity using Blow Count and SPT.

### RESEARCH EXPERIENCE

- Oct. 04 – Sep. 05     **Postdoctoral Research Fellow**  
Institute of Engineering Mechanics (IfM),  
University of Innsbruck,  
Austria, EU
- Jan. 99 – Aug. 01     **Research Assistant**  
Department of Civil & Structural Engineering,  
The University of Manchester Institute of Science and Technology (UMIST), Manchester,  
United Kingdom.  
Objectives: Evaluated Fracture Behaviour of Steel Structures especially Steel Connections  
subjected to Northridge Earthquake in 1994 and Obtained the Simplify Method.

### WORK EXPERIENCE

- May.03 – present     **Assistant Professor**  
Department of Civil Engineering, UbonRatchathani  
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- Feb. 96 – Apr. 03     **Lecturer**  
Department of Civil Engineering, UbonRatchathani  
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- Jan. 99 – Aug. 01     **Teaching Assistant (Finite Element Class)**  
Department of Civil & Structural Engineering,  
The University of Manchester Institute of Science and Technology (UMIST), Manchester,  
United Kingdom.
- Jan. 99 – Aug. 01     **Laboratory Demonstrator (Structural Engineering Lab)**  
Department of Civil & Structural Engineering,  
The University of Manchester Institute of Science and Technology (UMIST), Manchester,  
United Kingdom.

### COMPUTER SKILLS

- Operating systems: Window NT, Window 95/98, Window XP, Linux, UNIX
- Software: ABAQUS, LUSAS, Sap2000, FEAP, MS Word, MS Excel, MS Powerpoint

## SCHOLARSHIP/HONORS/AWARDS

Oct. 04 – Sep. 05	<b>Research Grant:</b> Assessment of dynamic behaviour and loading capacity of railway bridges: Inspection and analytical technique
Oct. 03 – Sep. 04	<b>Research Grant:</b> A study and development of windows base program of reliability analysis for assessing service life of cracked connections
Oct. 03 – Sep. 04	<b>Research Grant:</b> A comparative study on dynamic response of different floor types subjected to walking load, dancing load and running load
Jul. 03 – Jun. 05	<b>Research Grant:</b> Assessment of fracture in offshore structures subjected to wave loading (TRF Grant)
Oct. 02 – Sep. 03	<b>Research Grant:</b> Finite Element of Long-Span Concrete Floor Subjected to Walking Load.
Sep. 97 – Sep. 01	Thai Government Scholarship for M.Sc./Ph.D. in Structural Engineering.
Apr. 95 – Oct. 95	Ubonratchathani University Scholarship for the Final Year of Undergraduate Student at KKU, Khon Kaen, Thailand.

## SELECTED PUBLICATIONS/PRESENTATIONS

### TECHNICAL REPORT

- 1 P.S. Koutsourelakis, **K. Kuntiyawichai**, and G.I. Schuëller, (2005), "Effect of material uncertainties on fatigue life calculations of aircraft fuselages: a cohesive element model", Fortschritt-Berichte VDI Verlag, Germany
- 2 **K. Kuntiyawichai** (2005), "Quality assurance issues of structures (Literature review)", *Internal working report No.47-05*, Institute of Engineering Mechanics (IfM), University of Innsbruck, Austria.

### INTERNATIONAL JOURNAL

- 1 **K. Kuntiyawichai** and S. Chucheeprakul, (2005), "Assessment of through-wall crack in Minimum structures subjected to wave loading", *Engineering Structures* (In Press).
- 2 P.S. Koutsourelakis, **K. Kuntiyawichai**, and G.I. Schuëller, (2005), "Effect of material uncertainties on fatigue life calculations of aircraft fuselages: a cohesive element model", *Engineering Fracture Mechanics*, Vol.73 (9), pp 1202-1219.
- 3 **K. Kuntiyawichai** and F.M. Burdekin, (2003), "Damage assessment of structures under Earthquake dynamic loading using Fourier transformation", *International Journal of Materials and Structural Reliability*, Vol.1, No. 1, pp. 1-18.
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### NATIONAL JOURNAL

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- 4 Note Sangtian, Pattarawan Pansuwan, Nakharin Sanookpant and **Kittisak Kuntiyawichai** (2005), "Artificial Laterite", *KKU Engineering Journal*, vol.32, No. 4, pp. 578-584

#### INTERNATIONAL CONFERENCE PAPER

- 1 P.S. Koutsourelakis, **K. Kuntiyawichai**, and G. I. Schuëller, (2005), "Fatigue life calculations including the crack initiation phase and material uncertainties: a cohesive element model" 9<sup>th</sup> International Conference On Structural Safety and Reliability, ICOSSAR2005, Rome, Italy
- 2 **Kuntiyawichai K.**, Chucheeprakul S., Lee M.M.K., (2004), "Analysis of offshore structures subjected to various types of sea waves" 23<sup>rd</sup> International Conference on Offshore Mechanics and Arctic Engineering, Vancouver, Canada
- 3 W. Puatatsananon, **K. Kuntiyawichai**, (2003), "Service life assessment of cruciform connection using probabilistic approach" The Fourth Regional Symposium on Infrastructure Development in Civil Engineering, RSID4, Thailand
- 4 **K. Kuntiyawichai**, N. Sangtian, (2002), "Finite element study of long-span flat concrete floor subjected to walking load", International Conference on Structural Stability and Dynamics, ICSSD 2002, Singapore

#### NATIONAL CONFERENCE PAPER

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- 8 S. Bhokha, **K. Kuntiyawichai**, 2004, "Ministerial rule on building assessment under the building code B.E. 2543 and future changes in building regulations in thailand", The 9th National Convention on Civil Engineering 2003 at Cha-um, Petchaburi, pp. SIE112-17.
- 9 N. Sangtian, **K. Kuntiyawichai**, 2004, "Piezocone saturation related to pore water pressure response", The 9th National Convention on Civil Engineering 2003 at Cha-um, Petchaburi, pp. GTE183-186
- 10 S. Nilrat, W. Puatatsananon, **K. Kuntiyawichai**, 2004, "Vibration behaviour of long-span flat concrete floor subjected to human walking", The 9th National Convention on Civil Engineering 2003 at Cha-um, Petchaburi, pp. STR78-83.
- 11 J. Multongka, S. Seelachot, **K. Kuntiyawichai**, 2004, The 9th National Convention on Civil Engineering 2003 at Cha-um, Petchaburi, pp. STR71-77.
- 12 **K. Kuntiyawichai**, S. Bhokha, T. Tubkaew, (2003), "Damage detection of cracked concrete using wavelet transformation", The 2nd Seminar on Highway Engineering (Manus Corvanich) 2003, Bangkok, (in Thai), pp. 217-230.



- 13 **K. Kuntiyawichai**, S. Bhokha, T. Tubkaew, (2003), "Engineering guide on dynamic analysis of bridge structure subjected to moving load", The 2nd Seminar on Highway Engineering (Manus Corvanich) 2003, Bangkok (in Thai), pp. 231-242.
- 14 **K. Kuntiyawichai**, N. Sangtian, S. Kanarkard, (2002), "Dynamic behaviour of long-span flat concrete floor due to walking load", The 8th National Convention on Civil Engineering 2002 at Khon Kaen, pp. STR124-129.

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## EDUCATION

Aug. 98 – Aug. 03	<b>University of Texas at Austin, Texas, USA.</b> Ph.D., Structural Engineering Thesis: Dynamic Progressive Collapse of Frame Structures	Aug. 2003
Aug. 96 – Aug. 97	<b>Colorado State University, Colorado, USA.</b> M.Sc., Structural Engineering and Mechanics; Thesis: Design Oriented Equations for Tapered Columns	Aug. 1997
June 91 – Apr. 95	<b>Khon Kaen University (KKU), Khon Kaen, Thailand.</b> B.Eng. (1st-Class Honors), Civil Engineering.	Apr. 1995

## RESEARCH EXPERIENCE

Aug. 98 – Aug. 03	<b>Research Assistant</b> Department of Civil & Environmental Engineering, University of Texas at Austin, Texas, USA.
Aug. 96 – Aug. 97	<b>Research Assistant</b> Department of Civil & Environmental Engineering, Colorado State University, Colorado, USA.

## WORK EXPERIENCE

2003 – present	<b>Lecturer</b> Department of Civil Engineering, UbonRatchathani University, Warinchamrab, UbonRatchathani 34190, Thailand
2000-2002	<b>Teaching Assistant/Grader,</b> University of Texas at Austin, USA
1995-1996	<b>Lecturer</b> Department of Civil Engineering, UbonRatchathani University, Warinchamrab, UbonRatchathani 34190, Thailand
1994	<b>Engineer/Estimator,</b> Taisei (Thailand) Co., Ltd.

## SCHOLARSHIP/HONORS/AWARDS

2003	Departmental Fellowship, Department of Civil Engineering, The University of Texas at Austin
1995	First-Class Honors, Bachelor of Engineering in Civil Engineering, Khon Kaen University
1994	Outstanding Academic Performance Award, Faculty of Engineering, Khon Kaen University
1993	Outstanding Academic Performance Award, Faculty of Engineering, Khon Kaen University
1992	Outstanding Academic Performance Award, Faculty of Engineering, Khon Kaen University

## SELECTED PUBLICATIONS/PRESENTATIONS

### INTERNATIONAL JOURNAL

- 1 Kaewkulchai, G. and Williamson, E.B. (2004) "Beam Element Formulation and Solution Procedure for Dynamic Progressive Collapse Analysis." Computers and Structures, Elsevier Science, V.82, No.7-8, p.639-651.

#### INTERNATIONAL CONFERENCE PAPER

- 1 **Kaewkulchai, G., Kaewsena, N. and Phannikul, I.** (2005) "Elastic Buckling Capacity of Tapered Columns." Australian Structural Engineering Conference 2005, Newcastle, ASEC2005, Australia, Sep 11 – 14, 2005.
- 2 **Kaewkulchai, G. and Williamson, E.B.** (2003) "Progressive Collapse Behavior of Planar Frame Structures." Proceedings, Response of Structures to Extreme Loading Conference, Elsevier Science, Toronto, Canada, Aug 3-6, 2003.
- 3 **Kaewkulchai, G. and Williamson, E.B.** (2003) "Dynamic Behavior of Planar Frames during Progressive Collapse." Proceedings, The 16th Engineering Mechanics Conference, American Society of Civil Engineers, University of Washington, Seattle, Washington, USA, July 17-21, 2003.
- 4 **Williamson, E.B. and Kaewkulchai, G.** (2003) "Computational Modeling of Structural Collapse." The Fifth U.S.-Japan Workshop on Performance-Based Earthquake Engineering Methodology for Reinforced Concrete Building Structures, Japan, Sep 10-11, 2003.
- 5 **Kaewkulchai, G. and Williamson, E.B.** (2002) "Dynamic Progressive Collapse of Frame Structures." Proceedings, The 15th Engineering Mechanics Conference, American Society of Civil Engineers, Columbia University, New York, NY, USA, June 2-5, 2002.

#### NATIONAL CONFERENCE PAPERS

- 1 **Puatatsananon, W., Kuntiyawichai, K., Kaewkulchai, G. and Limkatanyu, S.** (2005) "Effects of statistical variability of applied stress range and size of initial cracks in fatigue life of cruciform connection." The 10th National Convention on Civil Engineering 2005 at Chonburi, Petchaburi, pp. STR33.
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- 3 **Kaewsena, N., Kaewkulchai, G., Phannikul, I. and Limkatanyu, S.** (2005) "Design-Oriented Equations for Buckling of Slender Tapered Columns." The 10th National Convention of Civil Engineering, NCCE10, Pattaya, Thailand, May 2-4, 2005 (in Thai).
- 4 **Kaewkulchai, G., Puatatsananon, W., Kuntiyawichai, K. and Limkatanyu, S.** (2004) "Progressive Collapse of Building Frames." The 3rd PSU-Engineering Conference, PEC3, Prince of Songkla University, Songkla, Thailand, December 8-9, 2004.
- 5 **Limkatanyu, S., Puatatsananon, W., Kuntiyawichai, K. and Kaewkulchai, G.** (2004) "Seismic Analysis of Reinforced Concrete Frames Including Reinforcement Slippage Effects." The 3rd PSU-Engineering Conference, PEC3, Prince of Songkla University, Songkla, Thailand, December 8-9, 2004.
- 6 **Kuntiyawichai, K., Kaewkulchai, G., Kaewsena, N., and Limkatanyu, S.** (2004) "Comparative Study of Seismic Design Codes." The 3rd PSU-Engineering Conference, PEC3, Prince of Songkla University, Songkla, Thailand, Dec 8-9, 2004 (in Thai).
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- 8 **Kuntiyawichai, K., Puatatsananon, W. and Kaewkulchai, G.** (2004). "Dynamic Behaviour of Through Truss Bridge (TT) under Passing Train." The 8th Annual National Symposium on Computational Science and Engineering, ANSCSE8, Suranaree University of Technology, Nakhonratchasima, Thailand, July 21-23, 2004.